4.1 Solutions to Exercises

1. Linear, because the average rate of change between any pair of points is constant.

3. Exponential, because the difference of consecutive inputs is constant and the ratio of consecutive outputs is constant.

5. Neither, because the average rate of change is not constant nor is the difference of consecutive inputs constant while the ratio of consecutive outputs is constant.

7. $f(x) = 11{,}000(1.085)^x$ You want to use your exponential formula f(x)= ab^x You know the initial value a is 11,000. Since b, your growth factor, is $b = 1 \pm r$, where r is the percent (written as a decimal) of growth/decay, $b = 1.085$. This gives you every component of your exponential function to plug in.

9. $f(x) = 23{,}900(1.09)^x$ $f(8) = 47{,}622$. You know the fox population is 23,900, in 2010, so that's your initial value. Since b, your growth factor is $b = 1 \pm r$, where r is the percent (written as a decimal) of growth/decay, $b = 1.09$. This gives you every component of your exponential function and produces the function$f(x) = 23{,}900(1.09)^x$. You're trying to evaluate the fox population in 2018, which is 8 years after 2010, the time of your initial value. So if you evaluate your function when $x = 8$, because $2018 - 2010 = 8$, you can estimate the population in 2018.

11. $f(x) = 32{,}500(.95)^x$ $f(12) = \$17{,}561.70$. You know the value of the car when purchased is 32,500, so that's your initial value. Since your growth factor is $b = 1 \pm r$, where r is the percent (written as a decimal) of growth/decay, $b = .95$ This gives you every component of your exponential function produces the function $f(x) = 32{,}500(.95)^x$. You're trying to evaluate the value of the car 12 years after it's purchased. So if you evaluate your function when $x = 12$, you can estimate the value of the car after 12 years.

13. We want a function in the form $f(x) = ab^x$. Note that $f(0) = ab^0 = a$; since (0, 6) is a given point, $f(0) = 6$, so we conclude $a = 6$. We can plug the other point (3, 750), into $f(x) = 6b^x$ to solve for b: $750 = 6(b)^3$. Solving gives $b = 5$, so $f(x) = 6(5)^x$.

15. We want a function in the form $f(x) = ab^x$. Note that $f(0) = ab^0 = a$; since (0, 2000) is a given point, $f(0) = 2000$, so we conclude $a = 2000$. We can plug the other point (2, 20) into $f(x) = 2000b^x$, giving $20 = 2000(b)^2$. Solving for *b*, we get $b = 0.1$, so $f(x) = 2000(.1)^x$.

17. $f(x) = 3(2)^x$ For this problem, you are not given an initial value, so using the coordinate points your given, $\left(-1, \frac{3}{2}\right), (3, 24)$ you can solve for b and then a. You know for the first coordinate point, $\left(\frac{3}{2}\right) = a(b)^{-1}$. You can now solve for a in terms of b: $\left(\frac{3}{2}\right) = \frac{a}{b} \rightarrow \left(\frac{3b}{2}\right) = a$. Once you know this, you can substitute $\left(\frac{3b}{2}\right) = a$, into your general equation, with your other coordinate point, to solve for b: $24 = \left(\frac{3b}{2}\right)(b)^3 \rightarrow 48 = 3b^4 \rightarrow 16 = b^4 \rightarrow b = 2$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points your given: $24 = a(2)^3 \rightarrow 24 = 8a \rightarrow a = 3$. So now that you've solved for a and b, you can come up with your general equation: $f(x) = 3(2)^x$.

19. $f(x) = 2.93(.699)^x$ For this problem, you are not given an initial value, so using the coordinate points you're given, $(-2,6), (3, 1)$ you can solve for b and then a. You know for the first coordinate point, $1 = a(b)^3$. You can now solve for a in terms of b: $\frac{1}{b^3} = a$. Once you know this, you can substitute $\frac{1}{b^3} = a$, into your general equation, with your other coordinate point, to solve for b: $6 = \frac{1}{b^3}(b)^{-2} \rightarrow 6b^5 = 1 \rightarrow b^5 = \frac{1}{6} \rightarrow b = .699$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points you're given: $6 = a(.699)^{-2} \rightarrow$ 6=2.047a $\rightarrow a = 2.93$. So now that you've solved for a and b, you can come up with your general equation: $f(x) = 2.93(.699)^x$

21. $f(x) = \frac{1}{8}(2)^x$ For this problem, you are not given an initial value, so using the coordinate points you're given, $(3,1), (5, 4)$ you can solve for b and then a. You know for the first coordinate point, $1 = a(b)^3$. You can now solve for a in terms of b: $1/b^3 = a$. Once you know this, you can substitute $\frac{1}{b^3} = a$, into your general equation, with your other coordinate point, to solve for b: $4 = \frac{1}{b^3}(b)^5 \rightarrow 4 = b^2 \rightarrow b = 2$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points your given: $1 = a(2)^3 \rightarrow 1 = 8a \rightarrow a = 1/8$. So now that you've solved for a and b, you can come up with your general equation: $f(x) = \frac{1}{8}(2)^x$

23. 33.58 milligrams. To solve this problem, you want to use the exponential growth/decay formula , $f(x) = a(b)^x$, to solve for b, your growth factor. Your starting amount is a, so a=100 mg. You are given a coordinate, (35,50), which you can plug into the formula to solve for b, your effective growth rate giving you your exponential formula $f(x) = 100(0.98031)^x$ Then you can plug in your $x = 54,$ to solve for your substance.

25. \$1,555,368.09 Annual growth rate: 1.39% To solve this problem, you want to use the exponential growth/decay formula f(x)=ab^x First create an equation using the initial conditions, the price of the house in 1985, to solve for a. You can then use the coordinate point you're given to solve for b. Once you've found a, and b, you can use your equation f(x)=$110{,}000(1.0139)^x$ to predict the value for the given year.

27. \$4,813.55 To solve this problem, you want to use the exponential growth/decay formula f(x)=ab^x First create an equation using the initial conditions, the value of the car in 2003, to solve for a. You can then use the coordinate point you're given to solve for b. Once you've found a, and b, you can use your equation f(x)=$38{,}000(.81333)^x$ to predict the value for the given year.

29. Annually: \$7353.84 Quarterly: \$47469.63 Monthly: \$7496.71 Continuously: \$7,501.44. Using the compound interest formula A(t)=$a(1+\frac{r}{K})^{Kt}$ you can plug in your starting amount,

\$4000 to solve for each of the three conditions, annually—$k = 1$, quarterly—$k = 4$, and monthly—$k = 12$. You then need to plug your starting amount, \$4000 into the continuous growth equation f(x)=ae^{rx} to solve for continuous compounding.

31. APY= .03034 ≈ 3.03% You want to use the APY formula $f(x) = (1 + \frac{r}{K})^{K}$-1 you are given a rate of 3% to find your r and since you are compounding quarterly K=4

33. $t = 7.4$ years To find out when the population of bacteria will exceed 7569 you can plug that number into the given equation as P(t) and solve for t. To solve for t, first isolate the exponential expression by dividing both sides of the equation by 1600, then take the ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for t.

35. (a) $w(t) = 1.1130(1.0464)^t$ For this problem, you are not given an initial value, since 1960 corresponds to 0, 1968 would correspond to 8 and so on, giving you the points $(8,1.60)$ $(16,2.30)$ you can use these points to solve for b and then a. You know for the first coordinate point, $1.60 = ab^8$. You can now solve for a in terms of b: $\frac{1.60}{b^8} = a$. Once you know this, you can substitute $\frac{1.60}{b^8} = a$, into your general equation, with your other coordinate point, to solve for b: $2.30 = \frac{1.60}{b^8}(b)^{16} \rightarrow 1.60b^8 = 2.30 \rightarrow b^8 = \frac{2.30}{1.60} \rightarrow b = 1.0464$. So you have now solved for b. Once you have done that you can solve for a, by using what you calculated for b, and one of the coordinate points you're given: $2.30 = a(1.0464)^{16} \rightarrow$ $2.30 = 2.0664a \rightarrow a = 1.1130$. So now that you've solved for a and b, you can come up with your general equation: $w(t) = 1.1130(1.0464)^t$

(b) \$1.11 using the equation you found in part a you can find w(0)

(c) The actual minimum wage is less than the model predicted, using the equation you found in part a you can find w(36) which would correspond to the year 1996

37. (a) 512 dimes the first square would have 1 dime which is 2^0 the second would have 2 dimes which is 2^1 and so on, so the tenth square would have 2^9 or 512 dimes

(b) 2^{n-1} if n is the number of the square you are on the first square would have 1 dime which is 2^{1-1} the second would have 2 dimes which is 2^{2-1} the fifteenth square would have 16384 dimes which is 2^{15-1}

(c) 2^{63}, 2^{64-1}

(d) 9,223,372,036,854,775,808 mm

(e) There are 1 million millimeters in a kilometer, so the stack of dimes is about 9,223,372,036,855 km high, or about 9,223,372 million km. This is approximately 61,489 times greater than the distance of the earth to the sun.

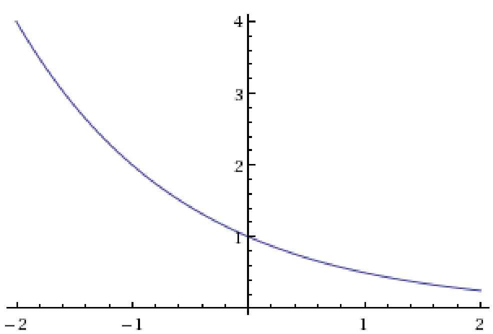
4.2 Solutions to Exercises

1. b 3. a 5. e

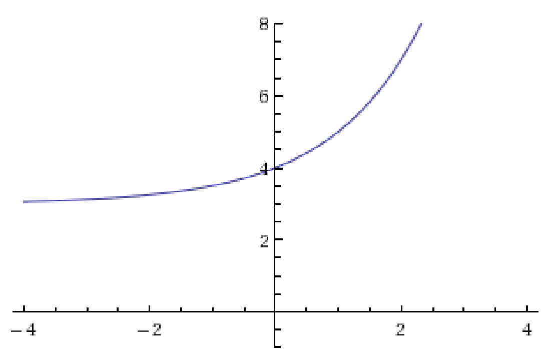
7. The value of b affects the steepness of the slope, and graph D has the highest positive slope it has the largest value for b.

9. The value of a is your initial value, when your $x = 0$. Graph C has the largest value for a.

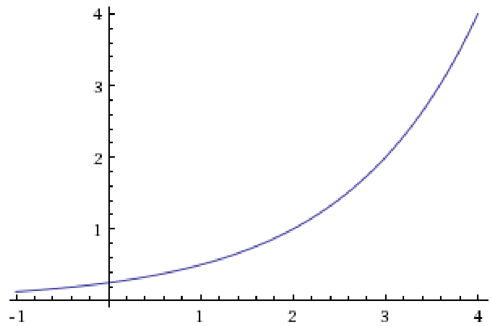
11. The function changes x to $-x$, which will reflect the graph across the y-axis.



13. The function will shift the function three units up.



15. The function will shift the function two units to the right.



17. $f(x) = 4^x + 4$ 19. $f(x) = 4^{(x+2)}$ 21. $f(x) = -4^x$

23. as $x \to \infty, f(x) \to -\infty$. When x is approaching $+\infty$, $f(x)$ becomes negative because 4^x is multiplied by a negative number.

    as $x \to -\infty, f(x) = -1$. As x approaches$-\infty$, $f(x)$ approaches 1, because $-5(4^{-x})$ will approach 0, which means $f(x)$ approaches -1 as it's shifted down one.

25. as $x \to \infty$, $f(x) \to -2$ As x approaches $+\infty$, $f(x)$ approaches -2, because $3\left(\frac{1}{2}\right)^x$ will approach 0, which means $f(x)$ approaches -2 as it's shifted down 2.

    as $x \to -\infty$, $f(x) \to +\infty$ because $\left(\frac{1}{2}\right)^{-x} = (2)^x$ so $f(x) \to \infty$.

27. as $x \to \infty$, $f(x) \to 2$ As x approaches $+\infty$, $f(x)$ approaches 2, because $3(4)^{-x}$ will approach 0, which means $f(x)$ approaches 2 as it's shifted up 2.

    as $x \to -\infty$, $f(x) \to \infty$ because $(4)^{-x} = \left(\frac{1}{4}\right)^x$ so $f(x) \to \infty$.

29. $f(x) = -2^{x+2} + 1$ flipped about the x-axis, horizontal shift 2 units to the left, vertical shift 1 unit up

31. $f(x) = -2^{-x} + 2$ flipped about the x-axis, flipped about the y-axis, vertical shift 2 units up

33. $f(x) = -2(3)^x + 7$ The form of an exponential function is $y = ab^x + c$. This equation has a horizontal asymptote at $x = 7$ so we know $c = 7$, you can also now solve for a and b by choosing two other points on the graph, in this case (0,5) an (1,1), you can then plug (0,5) into your general equation and solve for a algebraically, and then use your second point to solve for b.

35. $f(x) = 2\left(\frac{1}{2}\right)^x - 4$ The form of an exponential function is $y = ab^x + c$. This equation has a horizontal asymptote at $x = -4$ so we know $c = -4$, you can also now solve for a and b by choosing two other points on the graph, in this case (0,-2) an (-1,0), you can then plug (0,-2) into your general equation and solve for a algebraically, and then use your second point to solve for b.

4.3 Solutions to Exercises

1. $4^m = q$ use the inverse property of logs $\log_b c$=a is equivalent to b^a=c

3. $a^c = b$ use the inverse property of logs $\log_b c$=a is equivalent to b^a=c

5. $10^t = v$ use the inverse property of logs $\log_b c$=a is equivalent to b^a=c

7. $e^n = w$ use the inverse property of logs $\log_b c$=a is equivalent to b^a=c

9. $\log_4 y = x$ use the inverse property of logs b^a=c is equivalent to $\log_b c$=a

11. $\log_c k = d$ use the inverse property of logs b^a=c is equivalent to $\log_b c$=a

13. $\log b = a$ use the inverse property of logs b^a=c is equivalent to $\log_b c$=a

15. $\ln h = k$ use the inverse property of logs b^a=c is equivalent to $\log_b c$=a

17. $x = 9$ solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $3^2 = x$ then solve for x

19. $x = \frac{1}{8}$ solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $2^{-3} = x$ then solve for x

21. $x = 1000$ solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $10^3 = x$ then solve for x

23. $x = e^2$ solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $e^2 = x$

25. 2 solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $5^x = 25$ then solve for x

27. -3 solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $3^x = \frac{1}{27}$ then solve for x

29. $\frac{1}{2}$ solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $6^x = \sqrt{6}$ then solve for x

31. 4 solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $10^x = 10{,}000$ then solve for x

33. -3 solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $10^x = 0.001$ then solve for x

35. -2 solve using the inverse properties of logs to rewrite the logarithmic expression as the exponential expression $e^x = e^{-2}$ then solve for x

37. $x = -1.398$ use calculator

39. $x = 2.708$ use calculator

41. $x \approx 1.639$ Take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

43. $x \approx -1.392$ Take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

45. $x \approx 0.567$ Take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

47. $x \approx 2.078$ Take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

49. $x \approx 54.449$ First isolate the exponential expression by dividing both sides of the equation by 1000 to get it into b^a=c form, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

51. $x \approx 8.314$ First isolate the exponential expression by dividing both sides of the equation by 3 to get it into b^a=c form, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

53. $x \approx 13.412$ First isolate the exponential expression by dividing both sides of the equation by 50 to get it into b^a=c form, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

55. $x \approx .678$ First isolate the exponential expression by subtracting 10 from both sides of the equation and then dividing both sides by -8 to get it into b^a=c form, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for x.

57. $f(t) = 300e^{-.094t}$ You want to change from the form$f(t) = a(1+r)^t \; to \; f(t) = ae^{kt}$. From your initial conditions, you can solve for k by recognizing that, by using algebra, $(1+r) = e^k$. In this case $e^k = 0.91$ Then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, and then use algebra to solve for k. You then have all the pieces to plug into your continuous growth equation.

59. $f(t) = 10e^{.0392t}$ You want to change from the form $f(t) = a(1+r)^t \; to \; f(t) = ae^{kt}$. From your initial conditions, you can solve for k by recognizing that, by using algebra, $(1+r) = e^k$. In this case $e^k = 1.04$ Then take the log or ln of both sides of the equation, utilizing

the exponent property for logs to pull the variable out of the exponent, and then use algebra to solve for x. You then have all the pieces to plug into your continuous growth equation.

61. $f(t) = 150(1.062)^t$ You want to change from the form $f(t) = ae^{kt} to\ f(t) = a(1 + r)^t$. You can recognize that, by using algebra, $(1 + r) = e^k$. You can then solve for b, because you are given k, and you know that $b = (1 + r)$. Once you've calculated $b = 1.06184$, you have solved for all your variables, and can now put your equation into annual growth form.

63. $f(t) = 50(.988)^t$ You want to change from the form $f(t) = ae^{kt} to\ f(t) = a(1 + r)^t$. You can recognize that, by using algebra, $(1 + r) = e^k$. You can then solve for b, because you are given k, and you know that $b = (1 + r)$. Once you've calculated $b = .988072$, you have solved for all your variables, and can now put your equation into annual growth form.

65. 4.78404 years You want to use your exponential growth formula $y = ab^t$ and solve for t, time. You are given your initial value a= 39.8 million and we know that $b = (1 + r)$ you can solve for b using your rate, r=2.6% so b=1.026. You want to solve for t when f(t)=45 million so your formula is 45=39.8$(1.026)^t$. To solve for t, first isolate the exponential expression by dividing both sides of the equation by 39.8, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for t.

67. 74.2313 years You want to use your exponential growth formula $y = ab^t$ and first solve for b. You are given your initial value a=563,374 and you know that after 10 years the population grew to 608, 660 so you can write your equation 608,660=563,374$(b)^{10}$ and solve for b getting 1.00776. Now you want to find t when f(t)=1,000,000 so you can set up the equation 1,000,000=563,364$(1.00776)^t$. To solve for t, first isolate the exponential expression by dividing both sides of the equation by 563,364, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for t.

69. 34.0074 hrs You want to use your exponential decay formula $y = ab^t$ and first solve for b. You are given your initial value a=100mg and you know that after 4 hours the substance decayed

to 80mg so you can write your equation 80=$100(b)^4$ and solve for b getting .945742. Now you want to find t when f(t)=15 so you can set up the equation 15=$100(.945742)^t$. To solve for t, first isolate the exponential expression by dividing both sides of the equation by 100, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for t.

71. 13.5324 months You want to use your compound interest formula A(t)= $a(1+\frac{r}{k})^{kt}$ to solve for t when f(t)=1500. You are given your initial value a=1000, a rate of r=.03, and it compounds monthly so k=12. You can then write your equation as 1500=$1000(1+\frac{.03}{12})^{12t}$ and solve for t. To solve for t, first isolate the exponential expression by dividing both sides of the equation by 1000, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, then use algebra to solve for t.

4.4 Solutions to Exercises

1. $\log_3 4$ simplify using difference of logs property

3. $\log_3 7$ the -1 can be pulled inside the log by the exponential property to raise $\frac{1}{7}$ to the -1

5. $\log_3 5$ simplify using sum of logs property

7. $\log_7 2$ the $\frac{1}{3}$ can be pulled inside the log by the exponential property to raise 8 to the $\frac{1}{3}$

9. $\log(6x^9)$ simplify using sum of logs property

11. $\ln(2x^7)$ simplify using difference of logs property

13. $\log(x^2(x+1)^3)$ x can be raised to the 2nd power, and $(x+1)$ can be raised to the 3rd power via the exponential property, these two arguments can be multiplied in a single log via the sum of logs property

15. $\log\left(\frac{xz^3}{\sqrt{y}}\right)$ y can be raised to the -$\frac{1}{2}$ power, and z to the 3rd power via the exponential property, then these three arguments can be multiplied in a single log via the sum of logs property

17. $15\log(x) + 13\log(y) - 19\log(z)$ expand the logarithm by adding $\log(x^{15})$ and $\log(y^{13})$ (sum property) and subtracting $\log(z^{19})$ (difference property) then pull the exponent of each logarithm in front of the logs (exponential property)

19. $4\ln(b) - 2\ln(a) - 5\ln(c)$ expand the logarithm by adding $\ln(b^{-4})$ and $\ln(c^5)$ (sum property) and subtracting that from $\ln(a^{-2})$ (difference property) then pull the exponent of each logarithm in front of the logs (exponential property)

21. $\frac{3}{2}\log(x) - 2\log(y)$ expand the logarithm by adding $\log\left(x^{\frac{3}{2}}\right)$ and $\log\left(y^{\frac{-4}{2}}\right)$ (sum property) then pull the exponent of each logarithm in front of the logs (exponential property)

23. $\ln(y) + (\frac{1}{2}\ln(y) - \frac{1}{2}\ln(1-y))$ expand the logarithm by subtracting $\ln\left(y^{\frac{1}{2}}\right)$ and $\ln\left((1-y)^{\frac{1}{2}}\right)$ (difference property) and adding $\ln(y)$(sum property) then pull the exponent of each logarithm in front of the logs (exponential property)

25. $2\log(x)$+$3\log(y)$+$\frac{2}{3}\log(x)$+$\frac{5}{3}\log(y)$ expand the logarithm by adding $\log(x^2)$, $\log(y^3)$, $\log\left(x^{\frac{2}{3}}\right)$ and $\log\left(y^{\frac{5}{3}}\right)$ then pull the exponent of each logarithm in from of the logs (exponential property)

27. $x \approx -.7167$Take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, remembering to keep parenthesis on (4x-7) and (9x-6), and then use algebra to solve for x.

29. $x \approx -6.395$ divide both sides by 17 and $(1.16)^x$ using properties of exponents, then take the log or ln of both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent and then use algebra to solve for x

31. $t \approx 17.329$ divide both sides by 10 and $e^{(.12t)}$ using properties of exponents, then ln both sides of the equation, utilizing the exponent property for logs to pull the variable out of the exponent, remembering that ln(e)=1, and then use algebra to solve for t

33. $x = \frac{2}{7}$ rewrite as an exponential expression using the inverse property of logs and a base of 2 and then use algebra to solve for x

35. $x = \frac{1}{3e} \approx 0.1226$ subtract 3 from both sides of the equation and then divide both sides by 2, then rewrite as an exponential expression using the inverse property of logs and a base of e and then use algebra to solve for x

37. $x = \sqrt[3]{100} \approx 4.642$ rewrite as an exponential expression using the inverse property of logs and a base of 10 and then use algebra to solve for x

39. $x \approx 30.158$ combine the expression into a single logarithmic expression using the sum of logs property, then rewrite as an exponential expression using the inverse property of logs and a base of 10 and then use algebra to solve for x

41. $x = -\frac{26}{9} \approx -2.8889$ combine the expression into a single logarithmic expression using the difference of logs property, then rewrite as an exponential expression using the inverse property of logs and a base of 10 and then use algebra to solve for x

43. $x \approx -.872983$ combine the expression into a single logarithmic expression using the difference of logs property, then rewrite as an exponential expression using the inverse property of logs and a base of 6 and then use algebra to solve for x

45. $x = \frac{12}{11}$ combine the expression into a single logarithmic expression using the difference of logs property and the sum of logs property, then rewrite as an exponential expression using the inverse property of logs and a base of 10 and then use algebra to solve for x

47. $x = 10$ combine the expression into a single logarithmic expression using the difference of logs property and the sum of logs property, then rewrite as an exponential expression using the inverse property of logs and a base of 10 and then use algebra to solve for x

4.5 Solutions to Exercises

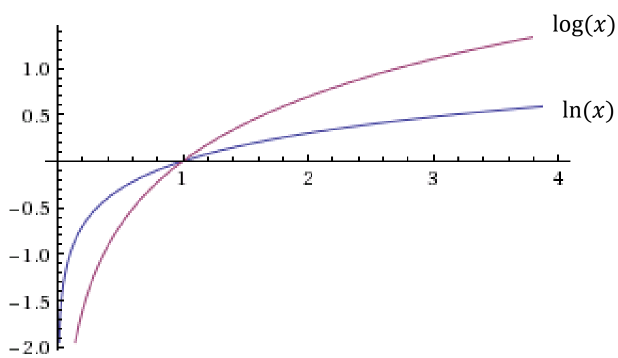
1. Domain: $x > 5$, vertical asymptote: $x = 5$.

3. Domain: $x < 3$, vertical asymptote: $x = 3$.

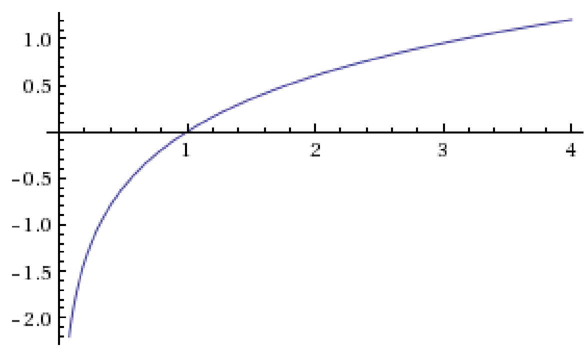
5. Domain: $x > -\frac{1}{3}$, vertical asymptote: $x = -\frac{1}{3}$.

7. Domain: $x < 0$, vertical asymptote: $x = 0$.

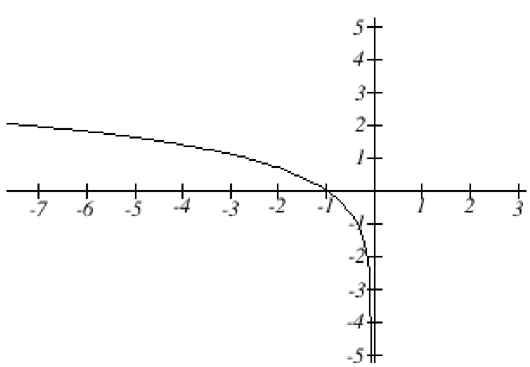
9.



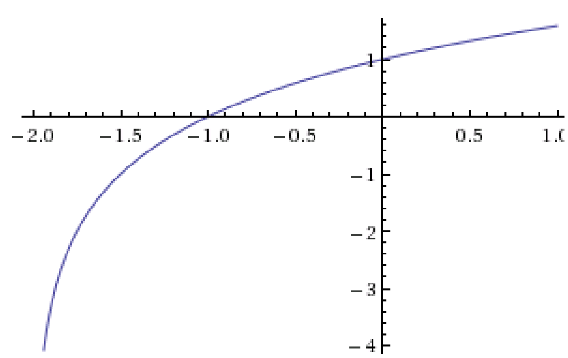
11.



13.



15.



17. $f(x) = 3.3219\log(1-x)$ Use the formula $f(x) = alog(x) + k$ and assume the function has a base of 10, first apply horizontal and vertical transformations if there are any, in this case a flip about the y-axis and a shift right 1, then to find the coefficient in front of the log plug in a given point (-1,1) in this case, and solve for a algebraically

19. $f(x) = -6.2877\log(x+4)$ Use the formula $f(x) = alog(x) + k$ and assume the function has a base of 10, first apply horizontal and vertical transformations if there are any, in this case a shift left 4, then to find the coefficient in front of the log plug in a given point (-1,-3) in this case, and solve for a algebraically

21. $f(x) = 4.9829\log(x+2)$ Use the formula $f(x) = alog(x) + k$ and assume the function has a base of 10, first apply horizontal and vertical transformations if there are any, in this case a shift left 2, then to find the coefficient in front of the log plug in a given point (2,3) in this case, and solve for a algebraically

23. $f(x) = -3.3219\log(5-x)$Use the formula $f(x) = alog(x) + k$ and assume the function has a base of 10, first apply horizontal and vertical transformations if there are any, in this case a flip about the y-axis and a shift right 5, then to find the coefficient in front of the log plug in a given point (0,-2) in this case, and solve for a algebraically

4.6 Solutions to Exercises

1. Letting *t* represent the number of minutes since the injection, we can model the number of milligrams remaining, $m(t)$, as $m(t) = ab^t$. Knowing that the initial number of milligrams is 13 tells us that $a = 13$, so $m(t) = 13b^t$. Substituting the values in the second sentence of the problem gives us an equation we can solve for *b*:

$$4.75 = 13b^{12}$$

$$\frac{4.75}{13} = b^{12}$$

$$b = \left(\frac{4.75}{13}\right)^{\frac{1}{12}} \approx 0.9195$$

Then $m(t) = 13(0.9257)^t$. We now use this model to find out the time at which 2 milligrams remain:

$$2 = 13(0.9195)^t$$
$$\frac{2}{13} = (0.9195)^t$$
$$\log\left(\frac{2}{13}\right) = \log((0.9195)^t)$$
$$\log\left(\frac{2}{13}\right) = t\log(0.9195)$$
$$t = \frac{\log\left(\frac{2}{13}\right)}{\log(0.9195)} \approx 22.3 \text{ minutes}$$

3. Using the form $h(t) = ab^t$, to find the number of milligrams after t years, where a is the initial amount of Radium-226 in milligrams, we first find b using the half-life of 1590 years:

$$0.5a = ab^{1590}$$
$$0.5 = b^{1590}$$
$$b = (0.5)^{\frac{1}{1590}}$$
$$b \approx 0.999564$$

We know also from the problem that $a = 200$. (We could have used this value when solving for the half-life, but it wasn't necessary.) Then $h(t) = 200(0.999564)^t$. To finish the problem, we compute the number of milligrams after 1000 years:

$$h(1000) = 200(0.999564)^{1000}$$
$$\approx 129.3$$

About 129.3 of Radium-226 milligrams remain after 1000 years.

5. Using the form $h(t) = ab^t$, where a is the initial amount in milligrams and t is time in hours, we first find b using the half-life of 10.4 hours:

$$0.5a = ab^{10.4}$$
$$0.5 = b^{10.4}$$
$$b = (0.5)^{\frac{1}{10.4}}$$
$$b \approx 0.935524$$

Then $h(t) = a \cdot 0.935528^t$. To find the original amount of the sample described:

$$2 = a \cdot 0.935524^{24}$$

$$2 = a \cdot 0.201983$$

$$a \approx 9.901810$$

(These numbers were obtained using longer decimals on the calculator instead of the rounded versions shown here.) In another 3 days, a total of 96 hours have elapsed:

$$h(96) = 9.901810(0.935524^{96})$$

$$h(96) \approx 0.016481$$

At this point, about 0.01648 mg of Erbium-165 remains.

7. 75.49 min. You are trying to solve for your half life. You first need to solve for your rate of decay, k, by using the information your given, and plugging it into your general equation, $\frac{1}{2}a = ae^{rt}$. By then taking the natural log of both sides you can solve for k, and with that given information solve for your half life.

9. 422.169 years ago. You are trying to solve for your time t when there is 60% of carbon present in living trees in your artifact. You first need to solve for your rate of decay, k, by using the information your given, and plugging it into your general equation, $\frac{1}{2}a = ae^{rt}$. By then taking the natural log of both sides you can solve for k, and with that given information solve for time.

11. (a) 23,700 bacteria.

(b) 14,962 bacteria. You want to use a formula for doubling time for this problem. You need to first solve for r, your continuous growth rate. Using the information you are given you can plug in your values from the original equation $n(t) = ae^{rt}$, then take the natural log of both sides to solve for r. Once you've solved for r you can use the equation to solve for amount of bacteria after your two given times.

13. (a) 611 bacteria. (b) 26.02 min. (c) 10,406 bacteria. (d) 107.057 min.

To solve part (a) of this problem, you need to first make two equations with the 2 points your given to solve for k algebraically by manipulating the functions so k is the only variable. Once you've solved for k, you can solve part (a), and then solve for the doubling time by using the general equation $n(t) = ae^{rt}$, and plugging in the information you're given, and solve part (b). You can then plug in given values to solve for (a), (b), (c), and (d).

15. Doubling time $t \approx 23.19$ years. We can use the compound interest equation from Section 4.1, $A(t) = a\left(1 + \frac{r}{k}\right)^{kt}$. To find the doubling time, since $A(t)$ represents the final amount after time t with initial amount a, we can modify this equation to $2a = a\left(1 + \frac{r}{k}\right)^{kt}$ which, since the a's cancel, equals $2 = \left(1 + \frac{r}{k}\right)^{kt}$. Plugging in the appropriate values gives $2 = \left(1 + \frac{0.03}{4}\right)^{4t}$. To solve for the doubling time t, we must take the log of both sides and use properties of logs.

17. 53.258 hours. For this problem, you can use the coordinate point your given, and plug in another value for t to get a second plotted point. Once you've done that, you can use the general formula $(t) = ab^t$, where you know your a, and can then plug in values to solve for b. Once you've done that you can find the doubling period by using the equation $2a = ab^t$, and taking the log of both sides.

19. (a) 134.2°F (b) 1.879 hours, or 112.7 min.

You want to use the formula for Newton's law of cooling, where T_s is the outside environment's temperature, a is a constant, and k is the continuous rate of cooling. You can first solve for a for evaluating $T(t)$ when $t = 0$. $165 = ae^{0k} + 75$. Solving for a gives $a = 90$. Use the temperature after half an hour to find k. (This solution is using hours as the units for t; you could also allow the units to be minutes, which would lead to a different value of k.)

$$145 = 90e^{0.5k} + 75$$

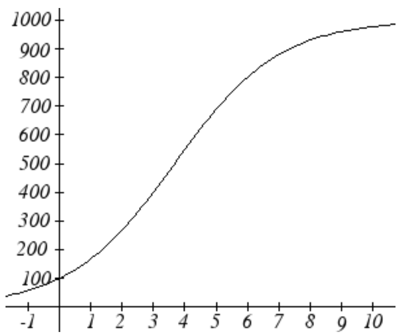
$$\frac{7}{9} = e^{0.5k}$$

$$\ln\left(\frac{7}{9}\right) = 0.5k$$

$$k \approx -0.5026$$

Then $T(t) \approx 90e^{-0.5026t} + 75$. Use this formula to solve (a) and (b), substituting $t = \frac{5}{6}$ hours for (a), and $T(t) = 110$ for (b). The steps to solve for t in (b) is similar to what we did to find k above.

21. (a)



$$P(t) = \frac{1000}{1 + 9e^{-0.6t}}$$

(b) 100 fish; plug in $t = 0$

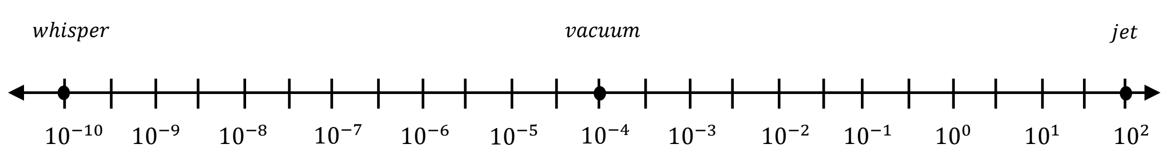
(c) 270 fish; plug in $t = 2$

(d) 7.34 years; let $P(t) = 900$ and solve for t.

23. 0.3162 To evaluate, you want to look at the value on the logarithmic scale, and then set that equal to $log(x)$. You can then rewrite that in exponential form to solve.

25. 31.623 To evaluate, you want to look at the value on the logarithmic scale, and then set that equal to $log(x)$. You can then rewrite that in exponential form to solve.

27.



29. $10^{4.8}$ or about 63,095.7 times greater. You want to plug in your values into the logarithmic form for each earthquake. Once you algebraically simplify these two equations, you want to change them from logarithmic to exponential form. You can then take the difference to see how many times more intense one of the earthquakes was than the other.

31. 5.8167. You know the magnitude of your original earthquake, which you can set to your equation, and then convert to exponential form. You can then multiply 750 by that exponential value, to solve for the magnitude of the second quake.

33. (a) 1,640,671 bacteria (b) 1.4 hours
(c) no, at small time values, the quantity is close enough, that you do not need to be worried.
(d) You should not be worried, because both models are within an order of magnitude after 6 hours. Given the equation, you can plug in your time to solve for after 1 hour. You can then use the doubling formula to solve for t, by taking the natural log of both sides.

35. (a) $M(p)$ is the top graph $H(p)$ is the bottom graph
 (b) 0.977507%
 (c) $H(t) = 32.4\%, M(t) = 95.2\%$
 (d) 20 torrs: 62.8%, 40 torrs: 20.6%, 60 torrs: 7.1%

You can evaluate which graph is which by plugging in values for t in each equation, and figuring out which graph is which. By plugging in 100 for p, you can solve for the level of oxygen saturation. You want to evaluate each equation at $p = 20$ to compare the level of hemoglobin. By following the definition of efficiency of oxygen, you want to evaluate both equations at $p = 20, 40, 60$, and then subtract $H(p)$ from $M(p)$.

37. (a) $C(t) = C_0 e^{.03466t}$ (b) $t = 433.87$ hours or $t \approx 18$ days
To find a formula, you can use the information that $C(t) = 2$ when $t = 20$, and $C_0 = 1$. Using this information you can take the natural log of both sides to solve for k. Once you've solved for k, you can come up with a general equation. For (b), you know $V = \frac{4}{3}\pi r^3$, where you can substitute your known radius in for r. You can use this information to solve for C_0. Once you've done this, you can set the equation $C(t) = 1$, and take the natural log of both sides to solve for t.

39. Since the number of termites and spiders are growing exponentially, we can model them as $T(t) = ab^t$ and $S(t) = cd^t$, respectively. We know the population when you move in (when $t = 0$) is 100. Then $100 = ab^0$, so $a = 100$. We also are given that there are 200 termites after 4 days, so $200 = 100b^4 \rightarrow 2 = b^4 \rightarrow b = 2^{1/4} \approx 1.1892$. Then our model is $T(t) =$

$100(1.1892)^t$. We can then use this formula to find $S(3) = \frac{1}{2}T(3) \approx 84$ and $S(8) = \frac{1}{4}T(8) \approx 100$. Then $84 = cd^3$ and $100 = cd^8$. Then:

$\frac{100}{84} = \frac{cd^8}{cd^3}$ → $d^5 \approx 1.1905$ → $d \approx 1.0355$

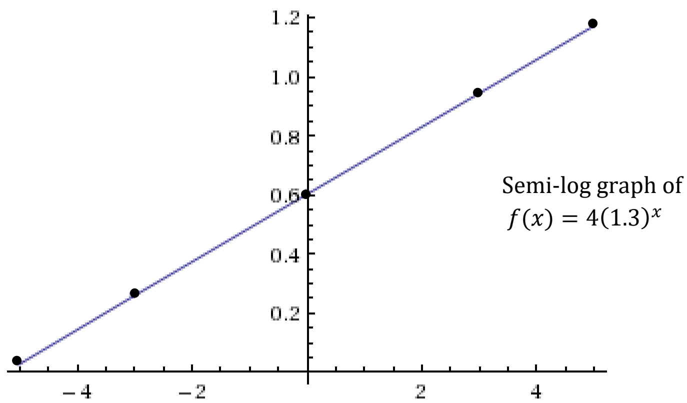
$84 = c(1.0355)^3$ → $c \approx 75.6535$

Since c represents the initial population of spiders, which should be a whole number, we'll round c to 76, so our model is $S(t) = 76(1.0355)^t$. To find when it triples, let $S(t) = 3 \cdot 76 = 228$. Then $228 = 76(1.0355)^t$ → $3 = (1.0355)^t$ → $t = \frac{\ln(3)}{\ln(1.0355)} \approx 31.5$ days.

4.7 Solutions to Exercises

1. Graph: We need to find 5 points on the graph, and then calculate the logarithm of the output value. Arbitrarily choosing 5 input values, we get ordered pairs $(x, y) = \left(x, \log\left(f(x)\right)\right)$

$\boldsymbol{x}$	$\boldsymbol{f(x)}$	$\boldsymbol{log\left(f(x)\right)}$	$\boldsymbol{(x, y)}$
-5	$4(1.3)^{-5}$ ≈ 1.07	log(1.07) = 0.029	(-5, 0.029)
-3	$4(1.3)^{-3}$ ≈ 1.82	0.260	(-3, 0.260)
0	$4(1.3)^0 = 4$	0.602	(0, 0.602)
3	$4(1.3)^3$ ≈ 8.78	0.943	(3, 0.943)
5	$4(1.3)^5$ ≈ 14.85	1.171	(5, 1.171)

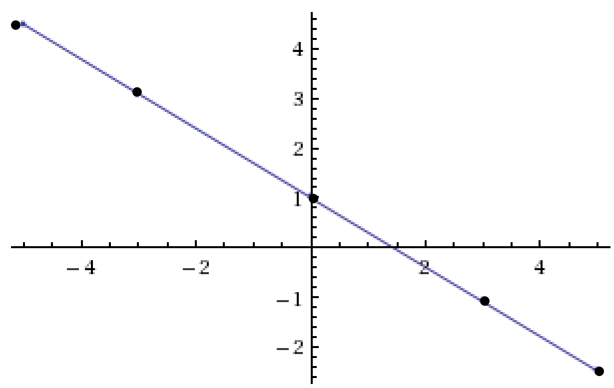


Equation: $\log(f(x)) = .4139x + .0021$. This is in the form $\log(f(x)) = \log(a) + x log(b)$, where $log(a)$ is the vertical intercept and $log(b)$ is the slope. You can solve for the y-intercept by setting $x = 0$ and then find another point to calculate the slope, and then put it into the form $log(f(x)) = mx + b$.

3. Graph: Refer to Problem (1)

Semi-log graph of
$f(x) = 10(0.2)^x$

$(\boldsymbol{x}, \boldsymbol{y})$
(-5, 4.49)
(-3, 3.09)
(0, 1)
(3, -1.09)
(5, -2.49)



Equation: $\log(f(x)) = -.699x + 1$. This is in the form $\log(f(x)) = \log(a) + x log(b)$, where $log(a)$ is the vertical intercept and $log(b)$ is the slope. You can solve for the y-intercept by setting $x = 0$ and then find another point to calculate the slope, and then put it into the form $log(f(x)) = mx + b$.

5. $f(x) = \frac{(1.648)^x}{e}$ You want to look at your graph to solve for your y-intercept, and then find another point on the graph so you can calculate the slope, to find the linear formula. Once you've found that, because this is a semi-natural log graph, you would want to rewrite it as the natural log exponential and then simplify.

7. $y(x) = 0.01(0.1)^x$ You want to look at your graph to solve for your y intercept, and then find another point on the graph so you can calculate the slope, to find the linear formula. Once you've found that, because this is a semi- log graph, you would want to rewrite it as the log exponential and then simplify.

9. $y(x) = 776.25(1.426)^x$ You first want to calculate every log(y) for your y values, and then from t here you can use technology to find a linear equation. Once you've found that, because this is a semi- log graph, you would want to rewrite it as the log exponential and then simplify.

11. $y(x) = 724.44(.738)^x$ You first want to calculate every log(y) for your y values, and then from t here you can use technology to find a linear equation. Once you've found that, because this is a semi- log graph, you would want to rewrite it as the log exponential and then simplify.

13. (a) $y = 54.954(1.054)^x$ (b) $204.65 billion in expenditures
For part (a), You first want to calculate every log(y) for your y values, and then from t here you can use technology to find a linear equation. Once you've found that, because this is a semi- log graph, you would want to rewrite it as the log exponential and then simplify. For part (b), your evaluating your function at $t = 25$, so plug that in for your equation to solve for y.

15. Looking at a scatter plot of the data, it appears that an exponential model is better. You first want to calculate every log(y) for your y values, and then from t you can use technology to find a linear equation. Once you've found that, because this is a semi-log graph, you want to rewrite it as the log exponential and then simplify, which gives $y(x) = 7.603(1.016)^x$. The evaluate your function at $t = 24$, so plug that in for your equation to get 11.128 cents per kilowatt hour.