# *Solutions Manual*
# *for*
# Precalculus

## An Investigation of Functions

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2nd Edition

**Solutions created at The Evergreen State College and Shoreline Community College**

## 1.1 Solutions to Exercises

1. (a) $f(40) = 13$, because the input 40 (in thousands of people) gives the output 13 (in tons of garbage)
   (b) $f(5) = 2$, means that 5000 people produce 2 tons of garbage per week.
3. (a) In 1995 (5 years after 1990) there were 30 ducks in the lake.
   (b) In 2000 (10 years after 1990) there were 40 ducks in the lake.

5. Graphs (a) (b) (d) and (e) represent *y* as a function of *x* because for every value of *x* there is only one value for *y*. Graphs (c) and (f) are not functions because they contain points that have more than one output for a given input, or values for *x* that have 2 or more values for *y*.

7. Tables (a) and (b) represent *y* as a function of *x* because for every value of *x* there is only one value for *y*. Table (c) is not a function because for the input x=10, there are two different outputs for *y*.

9. Tables (a) (b) and (d) represent *y* as a function of *x* because for every value of *x* there is only one value for *y*. Table (c) is not a function because for the input *x*=3, there are two different outputs for *y*.

11. Table (b) represents *y* as a function of *x* and is one-to-one because there is a unique output for every input, and a unique input for every output. Table (a) is not one-to-one because two different inputs give the same output, and table (c) is not a function because there are two different outputs for the same input x=8.

13. Graphs (b) (c) (e) and (f) are one-to-one functions because there is a unique input for every output. Graph (a) is not a function, and graph (d) is not one-to-one because it contains points which have the same output for two different inputs.

15. (a) $f(1) = 1$ (b) $f(3) = 1$

17. (a) $g(2) = 4$ (b) $g(-3) = 2$

19. (a) $f(3) = 53$ (b) $f(2) = 1$

21. $f(-2) = 4 - 2(-2) = 4 + 4 = 8, f(-1) = 6, f(0) = 4, f(1) = 4 - 2(1) = 4 - 2 = 2, f(2) = 0$

23. $f(-2) = 8(-2)^2 - 7(-2) + 3 = 8(4) + 14 + 3 = 32 + 14 + 3 = 49, f(-1) = 18, f(0) = 3, f(1) = 8(1)^2 - 7(1) + 3 = 8 - 7 + 3 = 4, f(2) = 21$

25. $f(-2) = -(-2)^3 + 2(-2) = -(-8) - 4 = 8 - 4 = 4, f(-1) = -(-1)^3 + 2(-1) = -(-1) - 2 = -1, f(0) = 0, f(1) = -(1)^3 + 2(1) = 1, f(2) = -4$

27. $f(-2) = 3 + \sqrt{(-2)+3} = 3 + \sqrt{1} = 3 + 1 = 4, f(-1) = \sqrt{2} + 3 \approx 4.41, f(0) = \sqrt{3} + 3 \approx 4.73, f(1) = 3 + \sqrt{(1)+3} = 3 + \sqrt{4} = 3 + 2 = 5, f(2) = \sqrt{5} + 3 \approx 5.23$

29. $f(-2) = \big((-2) - 2\big)\big((-2) + 3\big) = (-4)(1) = -4, f(-1) = -6, f(0) = -6, f(1) = \big((1) - 2\big)\big((1) + 3\big) = (-1)(4) = -4, f(2) = 0$

31. $f(-2) = \frac{(-2)-3}{(-2)+1} = \frac{-5}{-1} = 5, f(-1) = undefined, f(0) = -3, f(1) = -1, f(2) = -1/3$

33. $f(-2) = 2^{-2} = \frac{1}{2^2} = \frac{1}{4}, f(-1) = \frac{1}{2}, f(0) = 1, f(1) = 2, f(2) = 4$

35. Using $f(x) = x^2 + 8x - 4$: $f(-1) = (-1)^2 + 8(-1) - 4 = 1 - 8 - 4 = -11$; $f(1) = 1^2 + 8(1) - 4 = 1 + 8 - 4 = 5$.

(a) $f(-1) + f(1) = -11 + 5 = -6$

(b) $f(-1) - f(1) = -11 - 5 = -16$

37. Using $f(t) = 3t + 5$:

(a) $f(0) = 3(0) + 5 = 5$

(b) $3t + 5 = 0$

$t = -\frac{5}{3}$

39. (a) $y = x$ (iii. Linear)

(b) $y = x^3$ (viii. Cubic)

(c) $y = \sqrt[3]{x}$ (i. Cube Root)

(d) $y = \frac{1}{x}$ (ii. Reciprocal)

(e) $y = x^2$ (vi. Quadratic)

(f) $y = \sqrt{x}$ (iv. Square Root)

(g) $y = |x|$ (v. Absolute Value)

(h) $y = \frac{1}{x^2}$ (vii. Reciprocal Squared)

41. (a) $y = x^2$ (iv.)

(b) $y = x$ (ii.)

(c) $y = \sqrt{x}$ (v.)

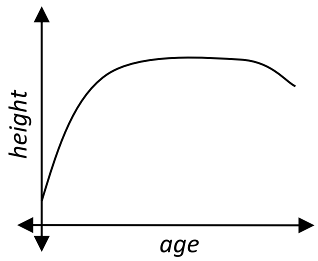
(d) $y = \frac{1}{x}$ (i.)

(e) $y = |x|$ (vi.)

(f) $y = x^3$ (iii.)

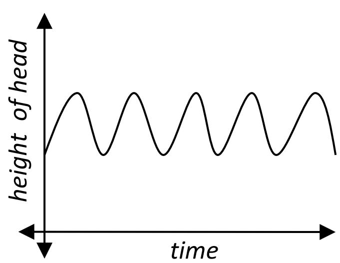
43. $(x-3)^2+(y+9)^2=(6)^2$ or $(x-3)^2+(y+9)^2=36$

45. (a)



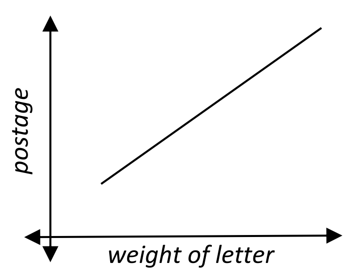

**Graph (a)**
At the beginning, as age increases, height increases. At some point, height stops increasing (as a person stops growing) and height stays the same as age increases. Then, when a person has aged, their height decreases slightly.

(b)




**Graph (b)**
As time elapses, the height of a person's head while jumping on a pogo stick as observed from a fixed point will go up and down in a periodic manner.

(c)




**Graph (c)**
The graph does not pass through the origin because you cannot mail a letter with zero postage or a letter with zero weight. The graph begins at the minimum postage and weight, and as the weight increases, the postage increases.

47. (a) $t$ (b) $x=a$ (c) $f(b)=0$ so $z=0$. Then $f(z)=f(0)=r$.

(d) $L=(c,t), K=(a,p)$

## 1.2 Solutions to Exercises

1. The domain is $[-5,3)$; the range is $[0,2]$

3. The domain is $2 < x \leq 8$; the range is $6 \leq y < 8$

5. The domain is $0 \leq x \leq 4$; the range is $0 \leq y \leq -3$

7. Since the function is not defined when there is a negative number under the square root, $x$ cannot be less than 2 (it can be equal to 2, because $\sqrt{0}$ is defined). So the domain is $x \geq 2$. Because the inputs are limited to all numbers greater than 2, the number under the square root will always be positive, so the outputs will be limited to positive numbers. So the range is $f(x) \geq 0$.

9. Since the function is not defined when there is a negative number under the square root, $x$ cannot be greater than 3 (it can be equal to 3, because $\sqrt{0}$ is defined). So the domain is $x \leq 3$. Because the inputs are limited to all numbers less than 3, the number under the square root will always be positive, and there is no way for 3 minus a positive number to equal more than three, so the outputs can be any number less than 3. So the range is $f(x) \leq 3$.

11. Since the function is not defined when there is division by zero, $x$ cannot equal 6. So the domain is all real numbers except 6, or $\{x | x \in \mathbb{R}, x \neq 6\}$. The outputs are not limited, so the range is all real numbers, or $\{y \in \mathbb{R}\}$.

13. Since the function is not defined when there is division by zero, $x$ cannot equal $-1/2$. So the domain is all real numbers except $-1/2$ , or $\{x | x \in \mathbb{R}, x \neq -1/2\}$. The outputs are not limited, so the range is all real numbers, or $\{y \in \mathbb{R}\}$.

15. Since the function is not defined when there is a negative number under the square root, $x$ cannot be less than $-4$ (it can be equal to $-4$, because $\sqrt{0}$ is defined). Since the function is also not defined when there is division by zero, $x$ also cannot equal 4. So the domain is all real numbers less than $-4$ excluding 4, or $\{x | x \geq -4, x \neq 4\}$. There are no limitations for the outputs, so the range is all real numbers, or $\{y \in \mathbb{R}\}$.

17. It is easier to see where this function is undefined after factoring the denominator. This gives $f(x) = \frac{x-3}{(x+11)(x-2)}$. It then becomes clear that the denominator is undefined when $x = -11$ and when $x = 2$ because they cause division by zero. Therefore, the domain is $\{x | x \in \mathbb{R}, x \neq -11, x \neq 2\}$. There are no restrictions on the outputs, so the range is all real numbers, or $\{y \in \mathbb{R}\}$.

19. $f(-1) = -4; f(0) = 6; f(2) = 20; f(4) = 24$

21. $f(-1) = -1; f(0) = -2; f(2) = 7; f(4) = 5$

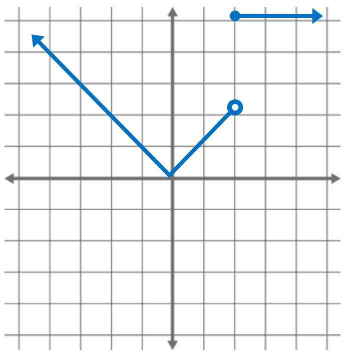
23. $f(-1) = -5; f(0) = 3; f(2) = 3; f(4) = 16$

25. $f(x) = \begin{cases} 2 & if \quad -6 \leq x \leq -1 \\ -2 & if \quad -1 < x \leq 2 \\ -4 & if \quad 2 < x \leq 4 \end{cases}$

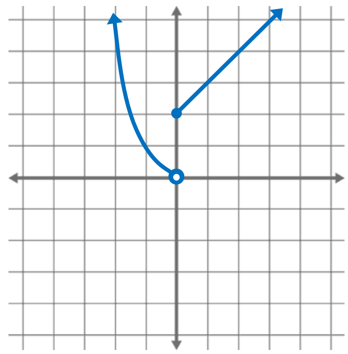
27. $f(x) = \begin{cases} 3 & if \quad x \leq 0 \\ x^2 & if \quad x > 0 \end{cases}$

29.. $f(x) = \begin{cases} \frac{1}{x} & if \quad x < 0 \\ \sqrt{x} & if \quad x \geq 0 \end{cases}$

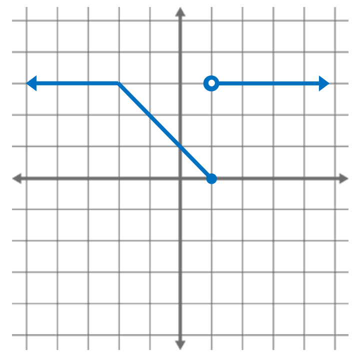
31.



33.



35.



## 1.3 Solutions to Exercises

1. (a) $\frac{249-243}{2002-2001} = \frac{6}{1} = 6$ million dollars per year

   (b) $\frac{249-243}{2004-2001} = \frac{6}{3} = 2$ million dollars per year

3. The inputs $x = 1$ and $x = 4$ produce the points on the graph: (4,4) and (1,5). The average rate of change between these two points is $\frac{5-4}{1-4} = \frac{1}{-3} = -\frac{1}{3}$.

5. The inputs $x = 1$ and $x = 5$ when put into the function $f(x)$ produce the points (1,1) and (5,25). The average rate of change between these two points is $\frac{25-1}{5-1} = \frac{24}{4} = 6$.

7. The inputs $x = -3$ and $x = 3$ when put into the function $g(x)$ produce the points (-3, -82) and (3,80). The average rate of change between these two points is $\frac{80-(-82)}{3-(-3)} = \frac{162}{6} = 27$.

9. The inputs $t = -1$ and $t = 3$ when put into the function $k(t)$ produce the points (-1,2) and (3,54.14$\overline{8}$). The average rate of change between these two points is $\frac{54.148-2}{3-(-1)} = \frac{52.148}{4} \approx 13$.

11. The inputs $x = 1$ and $x = b$ when put into the function $f(x)$ produce the points (1,-3) and $(b, 4b^2 - 7)$. *Explanation*: $f(1) = 4(1)^2 - 7 = -3, f(b) = 4(b)^2 - 7$. The average rate of

change between these two points is $\frac{(4b^2-7)-(-3)}{b-1} = \frac{4b^2-7+3}{b-1} = \frac{4b^2-4}{b-1} = \frac{4(b^2-1)}{b-1} = \frac{4(b+1)(b-1)}{(b-1)} = 4(b+1)$.

13. The inputs $x = 2$ and $x = 2 + h$ when put into the function $h(x)$ produce the points (2,10) and $(2 + h, 3h + 10)$. $Explanation$: $h(2) = 3(2) + 4 = 10, h(2 + h) = 3(2 + h) + 4 = 6 + 3h + 4 = 3h + 10$. The average rate of change between these two points is $\frac{(3h+10)-10}{(2+h)-2} = \frac{3h}{h} = 3$.

15. The inputs $t = 9$ and $t = 9 + h$ when put into the function $a(t)$ produce the points $(9, \frac{1}{13})$ and $\left(9 + h, \frac{1}{h+13}\right)$. $Explanation$: $a(9) = \frac{1}{9+4} = \frac{1}{13}, a(9 + h) = \frac{1}{(9+h)+4} = \frac{1}{h+13}$. The average rate of change between these two points is $\frac{\frac{1}{h+13}-\frac{1}{13}}{(9+h)-9} = \frac{\frac{1}{h+13}-\frac{1}{13}}{h} = \left(\frac{1}{h+13} - \frac{1}{13}\right)\left(\frac{1}{h}\right) = \frac{1}{h(h+13)} - \frac{1}{13h} = \frac{1}{h^2+13h} - \frac{1}{13h}\left(\frac{\frac{h}{13}+1}{\frac{h}{13}+1}\right)$ $(to\ make\ a\ common\ denominator) = \frac{1}{h^2+13h} - \left(\frac{\frac{h}{13}+1}{h^2+13h}\right) = \frac{1-\frac{h}{13}-1}{h^2+13h} = \frac{\frac{h}{13}}{h^2+13h} = \frac{h}{13}\left(\frac{1}{h^2+13h}\right) = \frac{h}{13(h^2+13h)} = \frac{h}{13h(h+13)} = \frac{h}{13(h+13)}$.

17. The inputs $x = 1$ and $x = 1 + h$ when put into the function $j(x)$ produce the points (1,3) and $(1 + h, 3(1 + h)^3)$. The average rate of change between these two points is $\frac{3(1+h)^3-3}{(1+h)-1} = \frac{3(1+h)^3-3}{h} = \frac{3(h^3+3h^2+3h+1)-3}{h} = \frac{3h^3+9h^2+9h+3-3}{h} = \frac{3h^3+9h^2+9h}{h} = 3h^2 + 9h + 9 = 3(h^2 + 3h + 3)$.

19. The inputs $x = x$ and $x = x + h$ when put into the function $f(x)$ produce the points $(x, 2x^2 + 1)$ and $(x + h, 2(x + h)^2 + 1)$. The average rate of change between these two points is $\frac{(2(x+h)^2+1)-(2x^2+1)}{(x+h)-x} = \frac{(2(x+h)^2+1)-(2x^2+1)}{h} = \frac{2(x+h)^2+1-2x^2-1}{h} = \frac{2(x+h)^2-2x^2}{h} = \frac{2(x^2+2hx+h^2)-2x^2}{h} = \frac{2x^2+4hx+2h^2-2x^2}{h} = \frac{4hx+2h^2}{h} = 4x + 2h = 2(2x + h)$.

21. The function is increasing (has a positive slope) on the interval $(-1.5,2)$, and decreasing (has a negative slope) on the intervals $(-\infty, -1.5)\ and\ (2, \infty)$.

23. The function is increasing (has a positive slope) on the intervals $(-\infty, 1)$ *and* $(3.25,4)$ and decreasing (has a negative slope) on the intervals $(1,2.75)$ *and* $(4, \infty)$.

25. The function is increasing because as $x$ increases, $f(x)$ also increases, and it is concave up because the rate at which $f(x)$ is changing is also increasing.

27. The function is decreasing because as $x$ increases, $h(x)$ decreases. It is concave down because the rate of change is becoming more negative and thus it is decreasing.

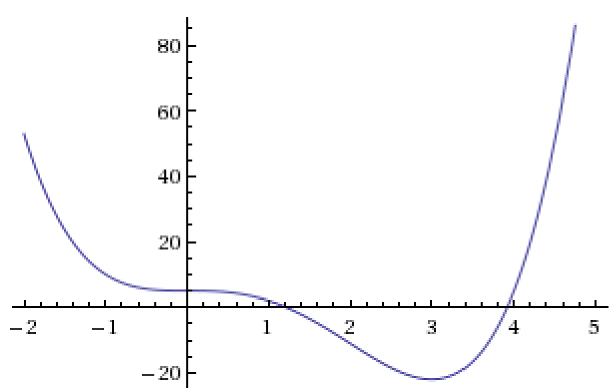
29. The function is decreasing because as $x$ increases, $f(x)$ decreases. It is concave up because the rate at which $f(x)$ is changing is increasing (becoming less negative).

31. The function is increasing because as $x$ increases, $h(x)$ also increases (becomes less negative). It is concave down because the rate at which $h(x)$ is changing is decreasing (adding larger and larger negative numbers).

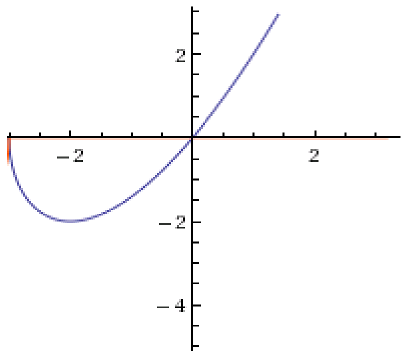
33. The function is concave up on the interval $(-\infty, 1)$, and concave down on the interval $(1, \infty)$. This means that $x = 1$ is a point of inflection (where the graph changes concavity).

35. The function is concave down on all intervals except where there is an asymptote at $x \approx 3$.

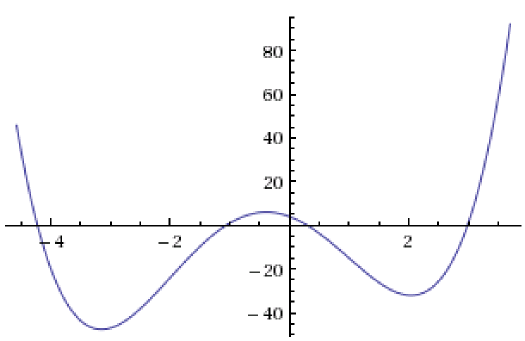
37. From the graph, we can see that the function is decreasing on the interval $(-\infty, 3)$, and increasing on the interval $(3, \infty)$. This means that the function has a local minimum at $x = 3$. We can estimate that the function is concave down on the interval $(0,2)$, and concave up on the intervals $(2, \infty)$and $(-\infty, 0)$. This means there are inflection points at $x = 2$ and $x = 0$.



39. From the graph, we can see that the function is decreasing on the interval $(-3, -2)$, and increasing on the interval $(-2, \infty)$. This means that the function has a local minimum at $x = -2$. The function is always concave up on its domain, (-3,∞). This means there are no points of inflection.




41. From the graph, we can see that the function is decreasing on the intervals $(-\infty, -3.15)$ *and* $(-0.38, 2.04)$, and increasing on the intervals $(-3.15, -0.38)$ *and* $(2.04, \infty)$ . This means that the function has local minimums at $x = -3.15$ *and* $x = 2.04$ and a local maximum at $x = -0.38$. We can estimate that the function is concave down on the interval $(-2,1)$, and concave up on the intervals $(-\infty, -2)$ *and* $(1, \infty)$. This means there are inflection points at $x = -2$ *and* $x = 1$.




## 1.4 Solutions to Exercises

1. $f(g(0)) = 4(7) + 8 = 26,\ g(f(0)) = 7 - (8)^2 = -57$

3. $f(g(0)) = \sqrt{(12) + 4} = 4,\ g(f(0)) = 12 - (2)^3 = 4$

5. $f(g(8)) = 4$ 7. $g(f(5)) = 9$

9. $f(f(4)) = 4$ 11. $g(g(2)) = 7$

13. $f(g(3)) = 0$ 15. $g(f(1)) = 4$

17. $f(f(5)) = 3$ 19. $g(g(2)) = 2$

21. $f(g(x)) = \frac{1}{\left(\frac{7}{x}+6\right)-6} = \frac{x}{7},\ g\big(f(x)\big) = \frac{7}{\left(\frac{1}{x-6}\right)} + 6 = 7x - 36$

23. $f(g(x)) = (\sqrt{x+2})^2 + 1 = x + 3, g(f(x)) = \sqrt{(x^2+1)+2} = \sqrt{(x^2+3)}$

25. $f(g(x)) = |5x+1|,\ g(f(x)) = 5|x| + 1$

27. $f(g(h(x))) = \left(\left(\sqrt{x}\right) - 6\right)^4 + 6$

29. b

31. (a) $r\big(V(t)\big) = \sqrt[3]{\frac{3(10+20t)}{4\pi}}$

(b) To find the radius after 20 seconds, we evaluate the composition from part (a) at $t = 20$.

$r\big(V(t)\big) = \sqrt[3]{\frac{3(10+20\cdot 20)}{4\pi}} \approx 4.609\ inches.$

33. $m\big(p(x)\big) = \left(\frac{1}{\sqrt{x}}\right)^2 - 4 = \frac{1}{x} - 4$. This function is undefined when the denominator is zero, or when $x = 0$. The inside function $p(x)$ is defined for $x > 0$. The domain of the composition is the most restrictive combination of the two: $\{x|x > 0\}$.

35. The domain of the inside function, $g(x)$, is $x \neq 1$. The composition is $f\big(g(x)\big) = \frac{1}{\frac{2}{x-1}+3}$.

Simplifying that, $f\big(g(x)\big) = \frac{1}{\frac{2}{x-1}+3} = \frac{1}{\frac{2}{x-1}+\frac{3(x-1)}{x-1}} = \frac{1}{\frac{2}{x-1}+\frac{3x-3}{x-1}} = \frac{1}{\frac{3x-1}{x-1}} = \frac{x-1}{3x-1}$. This function is undefined when the denominator is zero, giving domain $x \neq \frac{1}{3}$. Combining the two restrictions gives the domain of the composition: $\left\{x \middle| x \neq 1, x \neq \frac{1}{3}\right\}$.

In interval notation, $\left(-\infty, \frac{1}{3}\right) \cup \left(\frac{1}{3}, 1\right) \cup (1, \infty)$.

37. The inside function $f(x)$ requires $x - 2 \geq 0$, giving domain $x \geq 2$. The composition is $g(f(x)) = \frac{2}{\left(\sqrt{x-2}\right)^2 - 3} = \frac{2}{x-2-3} = \frac{2}{x-5}$, which has the restriction $x \neq 5$. The domain of the composition is the combination of these, so values larger than or equal to 2, not including 5: $\{x | 2 \leq x < 5 \text{ or } x > 5\}$, or $[2,5) \cup (5, \infty)$.

39. $f(x) = x^2,\ g(x) = x + 2$

41. $f(x) = \frac{3}{x},\ g(x) = x - 5$

43. $f(x) = 3 + x,\ g(x) = \sqrt{x - 2}$

45. (a) $f(x) = ax + b$, so $f(f(x)) = a(ax + b) + b$, which simplifies to $\mathrm{a}^2\mathrm{x} + 2\mathrm{b}$. $a$ and $b$ are constants, so $a^2$ and $2b$ are also constants, so the equation still has the form of a linear function.

(b) If we let $g(x)$ be a linear function, it has the form $g(x) = ax + b$. This means that $g(g(x)) = a(ax + b) + b$. This simplifies to $g(g(x)) = a^2x + ab + b$. We want $g(g(x))$ to equal $6x - 8$, so we can set the two equations equal to each other: $a^2x + ab + b = 6x - 8$. Looking at the right side of this equation, we see that the thing in front of the x has to equal 6. Looking at the left side of the equation, this means that $a^2 = 6$. Using the same logic, $ab + b = -8$. We can solve for , $a = \sqrt{6}$. We can substitute this value for $a$ into the second equation to solve for $b$: $(\sqrt{6})b + b = -8 \rightarrow b(\sqrt{6} + 1) = -8 \rightarrow b = -\frac{8}{\sqrt{6}+1}$. So, since $g(x) = ax + b$, $g(x) = \sqrt{6}x - \frac{8}{\sqrt{6}+1}$. Evaluating $g(g(x))$ for this function gives us 6x-8, so that confirms the answer.

47. (a) A function that converts seconds $s$ into minutes $m$ is $m = f(s) = \frac{s}{60}$. $C(f(s)) = \frac{70(\frac{s}{60})^2}{10+\left(\frac{s}{60}\right)^2}$ ; this function calculates the speed of the car in mph after $s$ seconds.

(b) A function that converts hours $h$ into minutes $m$ is $m = g(h) = 60h$. $C(g(h)) =$

$\frac{70(60h)^2}{10+(60h)^2}$ ; this function calculates the speed of the car in mph after $h$ hours.

(c) A function that converts mph $s$ into ft/sec $z$ is $z = v(s) = \left(\frac{5280}{3600}\right) s$ which can be reduced to $v(s) = \left(\frac{22}{15}\right) s$. $v(C(m)) = \left(\frac{22}{15}\right)\left(\frac{70m^2}{10+m^2}\right)$ ; this function converts the speed of the car in mph to ft/sec.

## 1.5 Solutions to Exercises

1. Horizontal shift 49 units to the right

3. Horizontal shift 3 units to the left

5. Vertical shift 5 units up

7. Vertical shift 2 units down

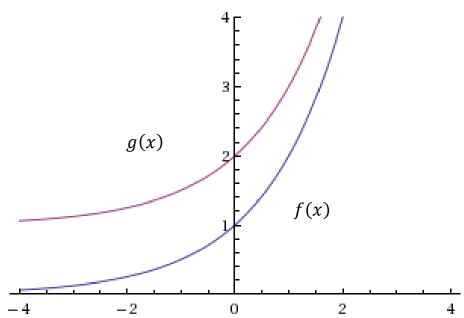
9. Horizontal shift 2 units to the right and vertical shift 3 units up

11. $f(x) = \sqrt{(x+2)} + 1$

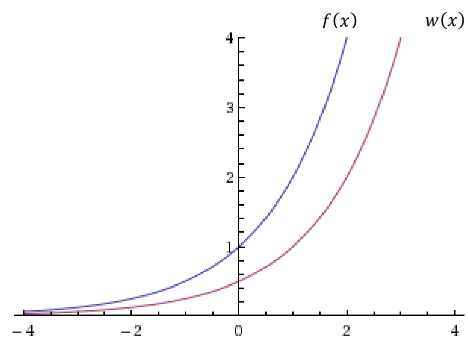
13. $f(x) = \frac{1}{(x-3)} - 4$

15. $g(x) = f(x-1)$, $h(x) = f(x) + 1$

17.

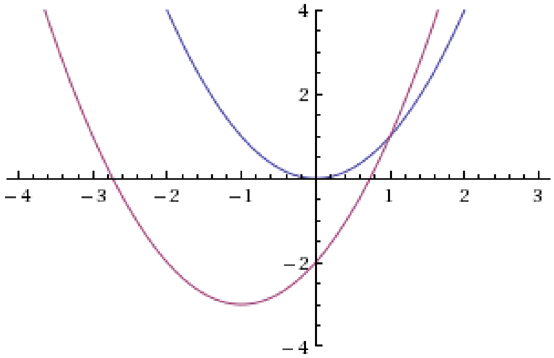



19.

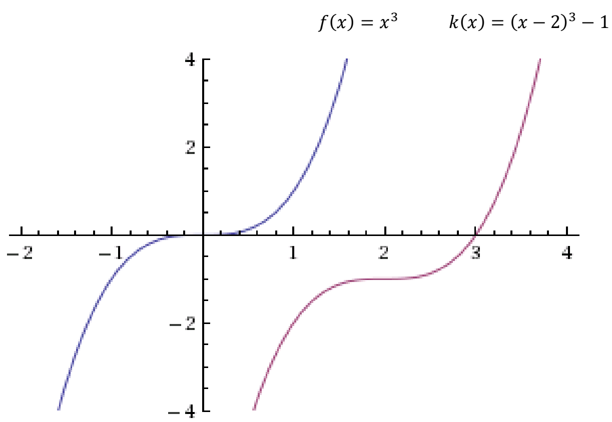



21. $f(t) = (t+1)^2 - 3$ as a transformation of $g(t) = t^2$

$f(t) = (t+1)^2 - 3$ $g(t) = t^2$




23. $k(x) = (x-2)^3 - 1$ as a transformation of $f(x) = x^3$

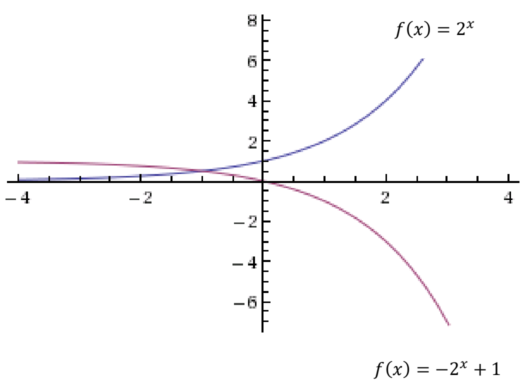



25. $f(x) = |x-3| - 2$

27. $f(x) = \sqrt{x+3} - 1$

29. $f(x) = -\sqrt{x}$

31.




33. (a) $f(x) = -6^{-x}$

(b) $f(x) = -6^{x+2} - 3$

35. $f(x) = -(x+1)^2 + 2$

37. $f(x) = \sqrt{-x} + 1$

39. (a) even (b) neither (c) odd

41. the function will be reflected over the x-axis

43. the function will be vertically stretched by a factor of 4

45. the function will be horizontally compressed by a factor of $\frac{1}{5}$

47. the function will be horizontally stretched by a factor of 3

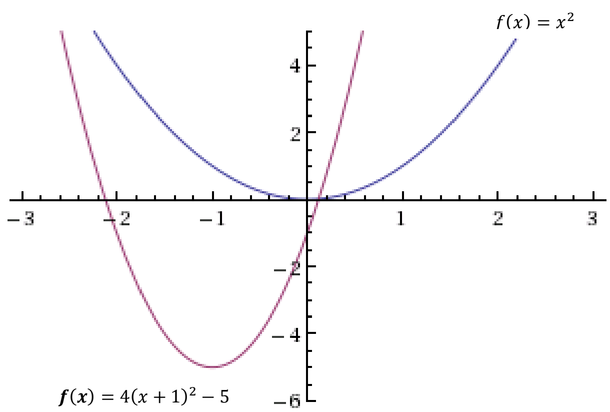
49. the function will be reflected about the y-axis and vertically stretched by a factor of 3

51. $f(x) = |-4x|$

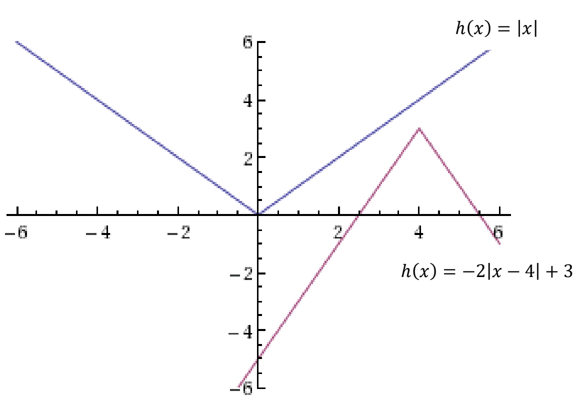
53. $f(x) = \frac{1}{3(x+2)^2} - 3$

55. $f(x) = (2[x-5])^2 + 1 = (2x-10)^2 + 1$

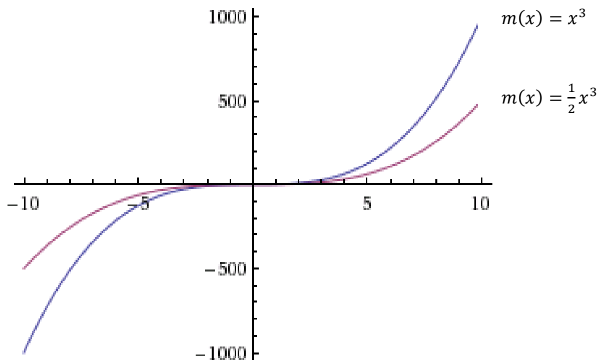
57. $f(x) = x^2$ will be shifted to the left 1 unit, vertically stretched by a factor of 4, and shifted down 5 units.



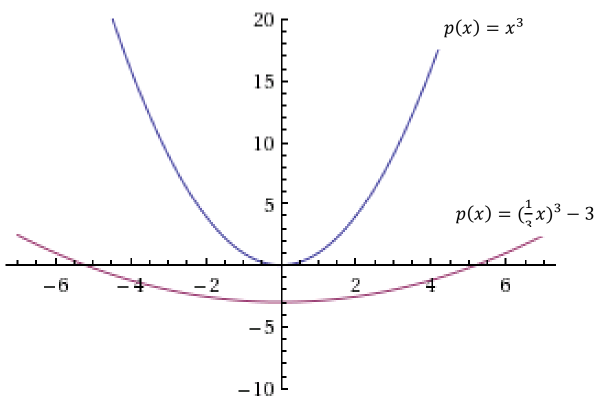

59. $h(x) = |x|$ will be shifted right 4 units vertically stretched by a factor of 2, reflected about the x-axis, and shifted up 3 units.



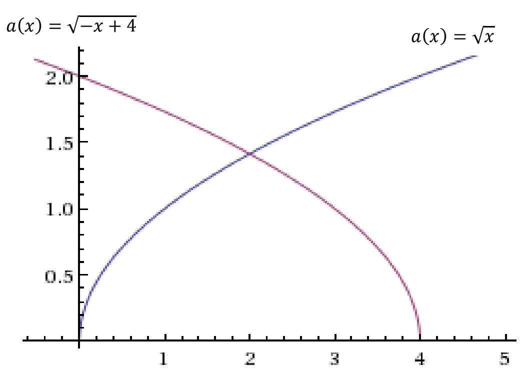
61. $m(x) = x^3$ will be vertically compressed by a factor of $\frac{1}{2}$.




63. $p(x) = x^2$ will be stretched horizontally by a factor of 3, and shifted down 3 units.




65. $a(x) = \sqrt{x}$ will be shifted left 4 units and then reflected about the y-axis.




67. the function is decreasing on the interval $x < -1$ and increasing on the interval $x > -1$

69. the function is decreasing on the interval $x \leq 4$

71. the function is concave up on the interval $x < -1$ and concave down on the interval $x > -1$

73. the function is always concave up.

75. $f(-x)$

77. $3f(x)$

79. $2f(-x)$

81. $2f\left(\frac{1}{2}x\right)$

83. $2f(x) - 2$

85. $-f(x) + 2$

87. $f(x) = -(x+2)^2 + 3$

89. $f(x) = \frac{1}{2}(x+1)^3 + 2$

91. $f(x) = \sqrt{2(x+2)} + 1$

93. $f(x) = -\frac{1}{(x-2)^2} + 3$

95. $f(x) = -|x+1| + 3$

97. $f(x) = -\sqrt[3]{x-2} + 1$

99. $f(x) = \begin{cases} (x+3)^2 + 1 & if\ x \leq -2 \\ -\frac{1}{2}|x-2| + 3 & if\ x > -2 \end{cases}$

101. $f(x) = \begin{cases} 1 & if\ x < -2 \\ -2(x+1)^2 + 4 & if\ -2 \leq x \leq 1 \\ \sqrt[3]{x-2} + 1 & if\ x > 1 \end{cases}$

103. (a) With the input in factored form, we first apply the horizontal compression by a factor of ½, followed by a shift to the right by three units. After applying the horizontal compression, the domain becomes $\frac{1}{2} \leq x \leq 3$. Then we apply the shift, to get a domain of $\left\{x \middle| 3\frac{1}{2} \leq x \leq 6\right\}$.

(b) Since these are horizontal transformations, the range is unchanged.

(c) These are vertical transformations, so the domain is unchanged.

(d) We first apply the vertical stretch by a factor of 2, followed by a downward shift of three units. After the vertical stretch, the range becomes $-6 \leq y \leq 10$. Next, we apply the shift to get the final domain $\{y|-9 \leq y \leq 7\}$.

(e) The simplest solution uses a positive value of *B*. The new domain is an interval of length one. Before, it was an interval of length 5, so there has been a horizontal compression by a factor of 1/5. Therefore, *B* = 5. If we apply this horizontal compression to the original domain, we get $\frac{1}{5} \leq x \leq \frac{6}{5}$. To transform this interval into one that starts at 8, we must add $7\frac{4}{5} = \frac{39}{5}$. This is our rightward shift, so $c = \frac{39}{5}$.

(f) The simplest solution uses a positive value of $A$. The new range is an interval of length one. The original range was an interval of length 8, so there has been a vertical compression by a factor of 1/8. Thus, we have $A = \frac{1}{8}$. If we apply this vertical compression to the original range we get $= \frac{3}{8} \leq y \leq \frac{5}{8}$. Now, in order to get an interval that begins at 0, we must add 3/8. This is a vertical shift upward, and we have $D = \frac{3}{8}$.

## 1.6 Solutions to Exercises

1. The definition of the inverse function is the function that reverses the input and output. So if the output is 7 when the input is 6, the inverse function $f^{-1}(x)$ gives an output of 6 when the input is 7. So, $f^{-1}(7) = 6$.

3. The definition of the inverse function is the function which reverse the input and output of the original function. So if the inverse function $f^{-1}(x)$ gives an output of $-8$ when the input is $-4$, the original function will do the opposite, giving an output of $-4$ when the input is $-8$. So $f(-8) = -4$.

5. $f(5) = 2$, so $\left(f(5)\right)^{-1} = (2)^{-1} = \frac{1}{2^1} = \frac{1}{2}$.

7. (a) $f(0) = 3$

(b) Solving $f(x) = 0$ asks the question: for what input is the output 0? The answer is $x = 2$. So, $f(2) = 0$.

(c) This asks the same question as in part (b). When is the output 0? The answer is $f^{-1}(0) = 2$.

(d) The statement from part (c) $f^{-1}(0) = 2$ can be interpreted as "in the original function $f(x)$, when the input is 2, the output is 0" because the inverse function reverses the original function. So, the statement $f^{-1}(x) = 0$ can be interpreted as "in the original function $f(x)$, when the input is 0, what is the output?" the answer is 3. So, $f^{-1}(3) = 0$.

9. (a) $f(1) = 0$ (b) $f(7) = 3$

(c) $f^{-1}(0) = 1$ (d) $f^{-1}(3) = 7$

| $x$ | 1 | 4 | 7 | 12 | 16 |
|---|---|---|---|---|---|

11.

| $f^{-1}(x)$ | 3 | 6 | 9 | 13 | 14 |
|---|---|---|---|---|---|

13. The inverse function takes the output from your original function and gives you back the input, or undoes what the function did. So if $f(x)$ adds 3 to $x$, to undo that, you would subtract 3 from $x$. So, $f^{-1}(x) = x - 3$.

15. In this case, the function is its own inverse, in other words, putting an output back into the function gives back the original input. So, $f^{-1}(x) = 2 - x$.

17. The inverse function takes the output from your original function and gives you back the input, or undoes what the function did. So if $f(x)$ multiplies 11 by $x$ and then adds 7, to undo that, you would subtract 7 from $x$, and then divide by 11. So, $f^{-1}(x) = \frac{x-7}{11}$.

19. This function is one-to-one and non-decreasing on the interval $x > -7$. The inverse function, restricted to that domain, is $f^{-1}(x) = \sqrt{x} - 7$.

21. This function is one-to-one and non-decreasing on the interval $x > 0$. The inverse function, restricted to that domain, is $f^{-1}(x) = \sqrt{x + 5}$.

23. (a) $f\big(g(x)\big) = \left(\left(\sqrt[3]{x+5}\right)\right)^3 - 5$, which just simplifies to $x$.
    (b) $g\big(f(x)\big) = \left(\sqrt[3]{(x^3 - 5) + 5}\right)$, which just simplifies to $x$.
    (c) This tells us that $f(x)$ and $g(x)$ are inverses, or, they undo each other.