1.7 Areas and Perimeters of Quadrilaterals

Learning Objective(s)

1 Calculate the perimeter of a polygon

2 Calculate the area of trapezoids and parallelograms

Introduction

We started exploring perimeter and area in earlier sections. In this section, we will explore perimeter in general, and look at the area of other quadrilateral (4 sided) figures.

Perimeter

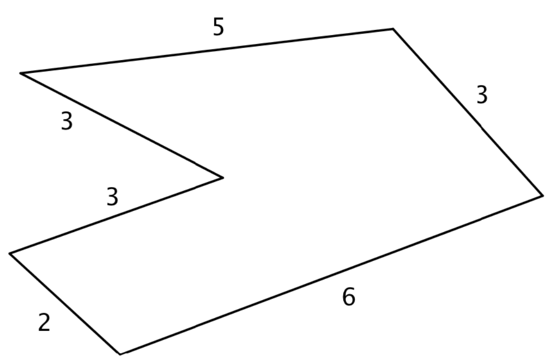
Objective **1**

The perimeter of a two-dimensional shape is the distance around the shape. You can think of wrapping a string around a rectangle. The length of this string would be the perimeter of the rectangle. Or walking around the outside of a park, you walk the distance of the park's perimeter. Some people find it useful to think "peRIMeter" because the edge of an object is its rim and peRIMeter has the word "rim" in it.

If the shape is a **polygon**, a shape with many sides, then you can add up all the lengths of the sides to find the perimeter. Be careful to make sure that all the lengths are measured in the same units. You measure perimeter in linear units, which is one dimensional. Examples of units of measure for length are inches, centimeters, or feet.

Example

Problem **Find the perimeter of the given figure. All measurements indicated are inches.**

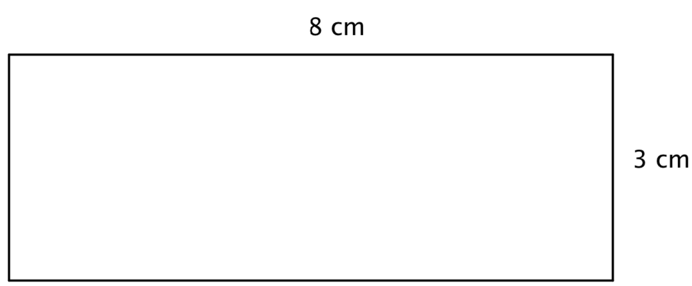


	$P = 5 + 3 + 6 + 2 + 3 + 3$	Since all the sides are measured in inches, just add the lengths of all six sides to get the perimeter.
Answer	$P = 22$ inches	Remember to include units.

This means that a tightly wrapped string running the entire distance around the polygon would measure 22 inches long.

Example		
Problem	**Find the perimeter of a triangle with sides measuring 6 cm, 8 cm, and 12 cm.**	
	$P = 6 + 8 + 12$	Since all the sides are measured in centimeters, just add the lengths of all three sides to get the perimeter.
Answer	P = 26 centimeters	

Sometimes, you need to use what you know about a polygon in order to find the perimeter. Let's look at the rectangle in the next example.

Example		
Problem	**A rectangle has a length of 8 centimeters and a width of 3 centimeters. Find the perimeter.** 	
	$P = 3 + 3 + 8 + 8$	Since this is a rectangle, the opposite sides have the same lengths, 3 cm. and 8 cm. Add up the lengths of all four sides to find the perimeter.
Answer	P = 22 cm	

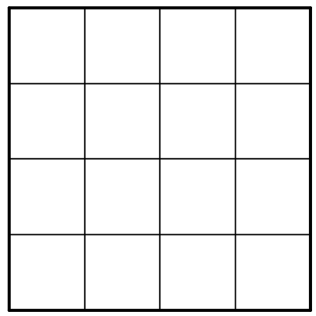
Notice that the perimeter of a rectangle always has two pairs of equal length sides. In the above example you could have also written $P = 2(3) + 2(8) = 6 + 16 = 22$ cm. The formula for the perimeter of a rectangle is often written as $P = 2l + 2w$, where l is the length of the rectangle and w is the width of the rectangle.

Area of Parallelograms

Objective **2**

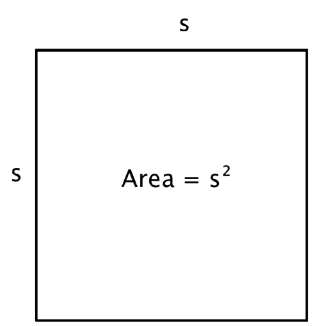
The area of a two-dimensional figure describes the amount of surface the shape covers. You measure area in square units of a fixed size. Examples of square units of measure are square inches, square centimeters, or square miles. When finding the area of a polygon, you count how many squares of a certain size will cover the region inside the polygon.

Let's look at a 4 x 4 square.



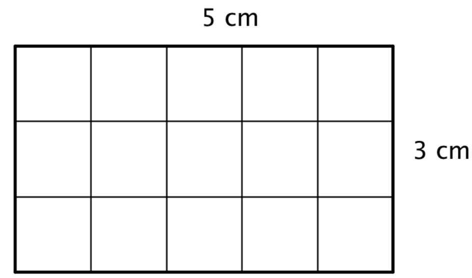
You can count that there are 16 squares, so the area is 16 square units. Counting out 16 squares doesn't take too long, but what about finding the area if this is a larger square or the units are smaller? It could take a long time to count.

Fortunately, you can use multiplication. Since there are 4 rows of 4 squares, you can multiply 4 • 4 to get 16 squares! And this can be generalized to a formula for finding the area of a square with any length, *s*: $Area = s \cdot s = s^2$.

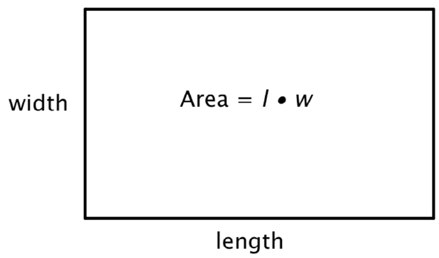


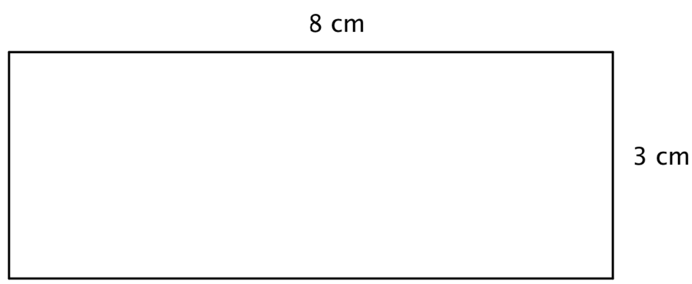
You can write "in^2" for square inches and "ft^2" for square feet.

To help you find the area of the many different categories of polygons, mathematicians have developed formulas. These formulas help you find the measurement more quickly than by simply counting. The formulas you are going to look at are all developed from the understanding that you are counting the number of square units *inside* the polygon. Let's look at a rectangle.



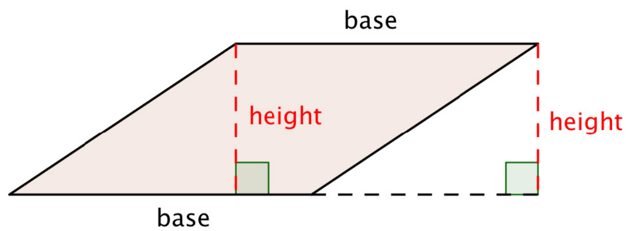
You can count the squares individually, but it is much easier to multiply 3 times 5 to find the number more quickly. And, more generally, the area of any rectangle can be found by multiplying *length* times *width.*

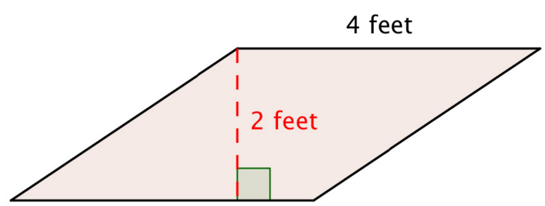


Example		
Problem	A rectangle has a length of 8 centimeters and a width of 3 centimeters. Find the area. 	
	$A = l \bullet w$	Start with the formula for the area of a rectangle, which multiplies the length times the width.
	$A = 8 \bullet 3$	Substitute 8 for the length and 3 for the width.
Answer	$A = 24$ cm^2	Be sure to include the units, in this case square cm.

It would take 24 squares, each measuring 1 cm on a side, to cover this rectangle.

The formula for the area of any parallelogram (remember, a rectangle is a type of parallelogram) is the same as that of a rectangle: *Area* = *l* • *w*. Notice in a rectangle, the length and the width are perpendicular. This should also be true for all parallelograms. *Base* (*b*) for the length (of the base), and *height* (*h*) for the width of the line perpendicular to the base is often used. So the formula for a parallelogram is generally written, *A* = *b* • *h.*



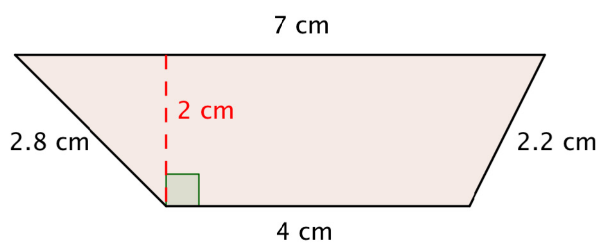
Example	
Problem	**Find the area of the parallelogram.**
	
$A = b \cdot h$	Start with the formula for the area of a parallelogram: *Area* = *base* • *height*.
$A = 4 \cdot 2$	Substitute the values into the formula.
$A = 8$	Multiply.
Answer	The area of the parallelogram is 8 ft^2.

Area of Trapezoids

Objective **2**

A trapezoid is a quadrilateral where two sides are parallel, but the other two sides are not. To find the area of a trapezoid, take the average length of the two parallel bases and multiply that length by the height: $A = \frac{(b_1 + b_2)}{2}h$.

An example is provided below. Notice that the height of a trapezoid will always be perpendicular to the bases (just like when you find the height of a parallelogram).

Example		
Problem	**Find the area of the trapezoid.**	
		
	$A = \frac{(b_1 + b_2)}{2}h$	Start with the formula for the area of a trapezoid.
	$A = \frac{(4+7)}{2} \cdot 2$ $A = \frac{11}{2} \cdot 2$ $A = 11$	Substitute 4 and 7 for the bases and 2 for the height, and find *A*.
Answer	The area of the trapezoid is 11 cm^2.	

Area Formulas

Use the following formulas to find the areas of different shapes.

square: $A = s^2$

rectangle: $A = l \cdot w$

parallelogram: $A = b \cdot h$

trapezoid: $A = \frac{(b_1 + b_2)}{2}h$

1.8.1 Properties and Laws of Whole Numbers

Learning Objective(s)
1 Simplify by using the addition property of 0.
2 Simplify by using the multiplication property of 1.
3 Identify and use the commutative law of addition.
4 Identify and use the commutative law of multiplication.
5 Identify and use the associative law of addition.
6 Identify and use the associative law of multiplication.

Introduction

Mathematics often involves simplifying numerical expressions. When doing so, you can use laws and properties that apply to particular operations. The multiplication property of 1 states that any number multiplied by 1 equals the same number, and the addition property of zero states that any number added to zero is the same number.

Two important laws are the commutative laws, which state that the order in which you add two numbers or multiply two numbers does not affect the answer. You can remember this because if you *commute* to work you go the same distance driving to work and driving home as you do driving home and driving to work. `You can move numbers around in addition and multiplication expressions because the order in these expressions does not matter.

You will also learn how to simplify addition and multiplication expressions using the associative laws. As with the commutative laws, there are associative laws for addition and multiplication. Just like people may associate with people in different groups, a number may *associate* with other numbers in one group or another. The associative laws allow you to place numbers in different groups using parentheses.

Addition and Multiplication Properties of 0 and 1

Objective **1, 2**

The addition property of 0 states that for any number being added to 0, the sum equals that number. Remember that you do not end up with zero as an answer – that only happens when you multiply. Your answer is simply the same as your original number.

Example	
Problem **62 + 0 = ?**	
62 + 0 = 62	Adding zero to 62 does not add any quantity to the sum, so the number remains 62.
Answer 62 + 0 = 62	

Self Check A
112 + 0 = ?

According to the **multiplication property of 1**, the product of 1 and any number results in that number. The answer is simply identical to the original number.

Example	
Problem **2,500 • 1 = ?**	
2,500 • 1 = 2,500	Multiplying 2,500 by 1 yields the same number.
Answer 2,500 • 1 = 2,500	

Self Check B
72,540 x 1 = ?

The Commutative Law of Addition

Objective 3

The **commutative law of addition** states that you can change the position of numbers in an addition expression without changing the sum. For example, 3 + 2 is the same as 2 + 3.

$$3 + 2 = 5$$
$$2 + 3 = 5$$

You likely encounter daily routines in which the order can be switched. For example, when you get ready for work in the morning, putting on your left glove and right glove is commutative. You could put the right glove on before the left glove, or the left glove on before the right glove. Likewise, brushing your teeth and combing your hair is commutative, because it does not matter which one you do first.

Remember that this law only applies to addition, and not subtraction. For example:

8 – 2 is not the same as 2 – 8.

Below, you will find examples of expressions that have been changed with the commutative law. Note that expressions involving subtraction cannot be changed.

Original Expression	Rewritten Expression
4 + 5	5 + 4
6 + 728	728 + 6
9 + 4 + 1	9 + 1 + 4
9 − 1	cannot be changed
72 − 10	cannot be changed
128 − 100	cannot be changed

You also will likely encounter real life routines that are not commutative. When preparing to go to work, putting on our clothes has to occur before putting on a coat. Likewise, getting in the car has to occur before putting the key in the ignition. In a store, you would need to pick up the items you are buying before proceeding to the cash register for checkout.

Example		
Problem		**Write the expression 10 + 25 in a different way, using the commutative law of addition, and show that both expressions result in the same answer.**
	10 + 25 = 35	Solving the problem yields an answer of 35.
	25 + 10	Using the commutative property, you can switch the 10 and the 25 so that they are in different positions.
	25 + 10 = 35	Adding 25 to 10 in this new order also yields 35.
Answer	10 + 25 = 35 and 25 + 10 = 35	

Self Check C
Rewrite 15 + 12 = 27 in a different way, using the commutative law of addition.

The Commutative Law of Multiplication

Objective 4

Multiplication also has a commutative law. The **commutative law of multiplication** states that when two or more numbers are being multiplied, their order can be changed without affecting the answer. In the example below, note that 5 multiplied by 4 yields the same result as 4 multiplied by 5. In both cases, the answer is 20.

$$5 \cdot 4 = 20$$
$$4 \cdot 5 = 20$$

This example shows how numbers can be switched in a multiplication expression.

Example	
Problem	**Write the expression 30 • 50 in a different way, using the commutative law of multiplication, and show that both expressions result in the same answer.**

30 • 50 = 1,500	Solving the problem yields an answer of 1,500.
50 • 30	Using the commutative law, you can switch the 30 and the 50 so that they are in different positions.
50 • 30 = 1,500	Multiplying 50 and 30 also yields 1,500.

Answer 50 • 30 and 30 • 50 = 1,500

Keep in mind that when you are using the commutative law, only the order is affected. The grouping remains unchanged.

Self Check D
Problem: Rewrite 52 • 46 in a different way, using the commutative law of multiplication.

The Associative Law of Addition

Objective **5**

Below are two ways of simplifying and solving an addition problem. Note that you can add numbers in any order. In the first example, 4 is added to 5 to make 9.

$$4 + 5 + 6 = 9 + 6 = 15$$

Here, the same problem is solved, but this time, 5 is added to 6 to make 11. Note that solving it this way yields the same answer.

$$4 + 5 + 6 = 4 + 11 = 15$$

The **associative law of addition** states that numbers in an addition expression can be regrouped using parentheses. You can remember the meaning of the associative law by remembering that when you *associate* with family members, friends, and co-workers, you end up forming groups with them. In the following expression, parentheses are used to group numbers together so that you know what to add first. Note that when parentheses are present, any numbers within parentheses are numbers you will add first. The expression can be re-written with different groups using the associative law.

$$(4 + 5) + 6 = 9 + 6 = 15$$

$$4 + (5 + 6) = 4 + 11 = 15$$

Here, it is clear that the parentheses do not affect the final answer, the answer is the same regardless of where the parentheses are.

Example	
Problem	**Rewrite (5 + 8) + 3 using the associative law of addition. Show that the rewritten expression yields the same answer.**
(5 + 8) + 3 = 13 + 3 = 16	The original expression yields an answer of 16.
5 + (8 + 3) = 5 + 11 = 16	Grouping 8 and 3 instead of 5 and 8 results in the same answer of 16.
Answer	(5 + 8) + 3 = 16 and 5 + (8 + 3) = 16

When rewriting an expression using the associative law, remember that you are regrouping the numbers and not reversing the order, as in the commutative law.

Self Check E
Rewrite 10 + (5 + 6) using the associative property.

The Associative Law of Multiplication

Objective 6

Multiplication has an associative law that works exactly the same as the one for addition. The **associative law of multiplication** states that numbers in a multiplication expression can be regrouped using parentheses. The following expression can be rewritten in a different way using the associative law.

$$(2 \cdot 3) \cdot 4 = 2 \cdot (3 \cdot 4).$$

Here, it is clear that the parentheses do not affect the final answer, the answer is the same regardless of where the parentheses are.

Example	
Problem	**Rewrite (10 • 200) • 24 using the associative law of multiplication, and show that the rewritten expression yields the same answer.**
(10 • 200) • 24 = 2000 • 24 = 48,000	The original expression yields an answer of 48,000.
10 • (200 • 24) = 10 • 4800 = 48,000	Grouping 200 and 24 instead of 10 and 200 results in the same answer of 48,000.
Answer	(10 • 200) • 24 = 48,000 and 10 • (200 • 24) = 48,000

When rewriting an expression using the associative law, remember that you are regrouping the numbers and not changing the order. Changing the order uses the commutative law.

Self Check F
Rewrite $8 \cdot (7 \cdot 6)$ using the associative property.

Commutative or Associative?

When an expression is being rewritten, you can tell whether it is being rewritten using the commutative or associative laws based on whether the order of the numbers change or the numbers are being regrouped using parentheses.

If an expression is rewritten so that the order of the numbers is changed, the commutative law is being used.

Example

Problem **$10 \cdot 2 = 20$ is rewritten as $2 \cdot 10 = 20$. Was this expression rewritten using the commutative law or the associative law?**

$10 \bullet 2 = 20$

$2 \bullet 10 = 20$

Rewriting the expression involves switching the order of the numbers. Therefore, the commutative law is being used.

Answer The commutative law is being used to rewrite the expression.

Remember that when you associate with friends and family, typically you are *grouping* yourself with other people. So, if numbers in an expression are regrouped using parentheses and the order of numbers remains the same, then the associative law is being used.

Example

Problem **$2 \cdot (4 \cdot 6) = 48$ is rewritten as $(2 \cdot 4) \cdot 6 = 48$. Was this expression rewritten using the commutative law or the associative law?**

2 • (4 • 6) = 48

(2 • 4) • 6 = 48

Regrouping using parentheses does not change the order of the numbers. Therefore, the associative law is being used.

Answer The associative law is being used to rewrite the expression.

Self Check G

12 • (6 • 2) = 144 is rewritten as (12 • 6) • 2 = 144. Was this expression rewritten using the commutative law or the associative law?

If there are absolutely no parentheses in a problem that is being rewritten, you can assume the associative law is not being used.

Self Check H

17 • 3 = 51 is rewritten as 3 • 17 = 51. Was this expression rewritten using the commutative law or associative law?

Using the Associative and Commutative Laws

The associative and commutative laws are useful when you have an expression with only addition. Using the commutative law, the numbers can be reordered so that the numbers that are easiest to add are next to each other, and using the associative law, you can group them in any way.

For example, here are some of the ways we can add 6 + 5 + 4 using the associative and commutative laws. Note that the answer is always the same.

(6 + 5) + 4 = 11 + 4 = 15	grouping 6 and 5 to add first
(5 + 6) + 4 = 11 + 4 = 15	reordering 6 and 5
5 + (6 + 4) = 5 + 10 = 15	grouping 6 and 4 to add first
6 + (5 + 4) = 6 + 9 = 15	grouping 5 and 4 to add first
6 + (4 + 5) = 6 + 9 = 15	reordering 4 and 5
(6 + 4) + 5 = 10 + 5 = 15	grouping 6 and 4 to add first

Example	
Problem	**Write the expression 13 + 28 + 7 a different way to make it easier to simplify. Then simplify.**
13 + 28 + 7	
13 + 7 + 28	Using the commutative property, reorder the numbers 7 and 28 since 13 + 7 is easier to add than 13 + 28.
20 + 28	Using the associative property, group the 13 and 7 together and add them first.
48	Add 20 and 28.
Answer	13 + 28 + 7 = 13 + 7 + 28 = 48

Sometimes the commutative and associative laws can make the problem easy enough to do in your head.

Example	
Problem	**Jim is buying 8 pears, 7 apples, and 2 oranges. He decided the total number of fruit is 8 + 7 + 2. Use the commutative property to write this expression in a different way. Then find the total.**
8 + 7 + 2	
8 + 2 + 7	Using the commutative property, reorder 2 and 7.
10 + 7	Using the associative property, group the 8 and 2 together and add them first.
17	Add 10 and 7.
Answer	8 + 7 + 2 = 8 + 2 + 7 = 17

This also works when you are multiplying more than two numbers. You can use the commutative and associative laws freely if the expression involves only multiplication.

Example	
Problem	**There are 2 trucks in a garage, and each truck holds 60 boxes. There are 5 laptop computers in each box. Find the number of computers in the garage.**
2 • 60 • 5	In order to find the answer, you need to multiply the number of trucks times the number of boxes in each truck, and, then by the number of computers in each box.
2 • 5 • 60	Using the commutative property, reorder the 5 and the 60. Now you can multiply 2 • 5 first.
10 • 60	Using the associative property, multiply the 2 and the 5, 2 • 5 = 10.
600	Now it's easier to multiply 10 and 60 to get 600.
Answer	There are 600 computers in the garage.

Summary

The addition property of 0 states that for any number being added to zero, the sum is the same number. The multiplication property of 1 states that for any number multiplied by one, that answer is that same number. Zero is called the additive identity, and one is called the multiplicative identity.

When you rewrite an expression by a commutative law, you change the order of the numbers being added or multiplied. When you rewrite an expression using an associative law, you group a different pair of numbers together using parentheses.

You can use the commutative and associative laws to regroup and reorder any number in an expression that involves only addition. You can also use the commutative and associative laws to regroup and reorder any number in an expression that involves only multiplication.

1.8.1 Self Check Solutions

Self Check A

112 + 0 = ?

112

Adding zero to a number does not change a number.

Self Check B

72,540 x 1 = ?

72,540

Multiplying any number by 1 yields the same number, which is in this case 72,540.

Self Check C
Rewrite 15 + 12 = 27 in a different way, using the commutative law of addition.

12 + 15 = 27
The commutative law lets you change the order of the numbers being added.

Self Check D
Problem: Rewrite 52 • 46 in a different way, using the commutative law of multiplication.

46 • 52
The order of numbers is reversed, and the same two numbers are multiplied.

Self Check E
Rewrite 10 + (5 + 6) using the associative property.

(10 + 5) + 6
Here, the numbers are regrouped. Now 10 and 5 are grouped in parentheses instead of 5 and 6.

Self Check F
Rewrite 8 • (7 • 6) using the associative property.

(8 • 7) • 6
Here, the numbers are regrouped. Now 8 and 7 are grouped in parentheses instead of 7 and 6.

Self Check G
12 • (6 • 2) = 144 is rewritten as (12 • 6) • 2 = 144. Was this expression rewritten using the commutative law or the associative law?

associative law
The numbers are being regrouped using parentheses and the order of numbers does not change.

Self Check H
17 • 3 = 51 is rewritten as 3 • 17 = 51. Was this expression rewritten using the commutative law or associative law?

commutative law
The order of numbers is being switched, which shows that the commutative law is being used.

1.8.2 The Distributive Property

Learning Objective(s)
1 Simplify using the distributive property of multiplication over addition.
2 Simplify using the distributive property of multiplication over subtraction.

Introduction

The distributive property of multiplication is a very useful property that lets you simplify expressions in which you are multiplying a number by a sum or difference. The property states that the product of a sum or difference, such as 6(5 – 2), is equal to the sum or difference of the products – in this case, 6(5) – 6(2).

Remember that there are several ways to write multiplication. 3 x 6 = 3(6) = 3 • 6.
3 • (2 + 4) = 3 • 6 = 18.

Distributive Property of Multiplication

Objective **1**

The **distributive property of multiplication over addition** can be used when you multiply a number by a sum. For example, suppose you want to multiply 3 by the sum of 10 + 2.

$$3(10 + 2) = ?$$

According to this property, you can add the numbers and then multiply by 3.
3(10 + 2) = 3(12) = 36. Or, you can first multiply each addend by the 3. (This is called **distributing** the 3.) Then, you can add the products.

$$\mathbf{3\,(10 + 2) = 3(10) + 3(2)}$$

The multiplication of 3(10) and 3(2) will each be done before you add.
3(10) + 3(2) = 30 + 6 = 36. Note that the answer is the same as before.

You probably use this property without knowing that you are using it. When a group (let's say 5 of you) order food, and order the same thing (let's say you each order a hamburger for $3 each and a coke for $1 each), you can compute the bill (without tax) in two ways. You can figure out how much each of you needs to pay and multiply the sum times the number of you. So, you each pay (3 + 1) and then multiply times 5. That's 5(3 + 1) = 5(4) = 20. Or, you can figure out how much the 5 hamburgers will cost and the 5 cokes and then find the total. That's 5(3) + 5(1) = 15 + 5 = 20. Either way, the answer is the same, $20.

The two methods are represented by the equations below. On the left side, we add 10 and 2, and then multiply by 3. The expression is rewritten using the distributive property on the right side, where we distribute the 3, then multiply each by 3 and add the results. Notice that the result is the same in each case.

$$3(10+2)=3(10)+3(2)$$
$$3(12)=30+6$$
$$36=36$$

The same process works if the 3 is on the other side of the parentheses, as in the example below.

(10 + 2) 3 = (10)3 + (2)3

Example	
Problem	**Rewrite the expression 5(8 + 4) using the distributive property of multiplication over addition. Then simplify the result.**
5 (8 + 4) = 5(8) + 5(4)	In the original expression, the 8 and the 4 are grouped in parentheses. Using arrows, you can see how the 5 is distributed to each addend. The 8 and 4 are each multiplied by 5.
40 + 20 = 60	The resulting products are added together, resulting in a sum of 60.
Answer	5(8 + 4) = 5(8) + 5(4) = 60

Self Check A
Rewrite the expression 30(2 + 4) using the distributive property of addition.

Distributive Property of Multiplication over Subtraction

Objective **2**

The **distributive property of multiplication over subtraction** is like the distributive property of multiplication over addition. You can subtract the numbers and then multiply, or you can multiply and then subtract as shown below. This is called “distributing the multiplier.”

$$5\,(6 - 3) = 5(6) - 5(3)$$

The same number works if the 5 is on the other side of the parentheses, as in the example below.

$$(6 - 3)\,5 = (6)5 - (3)5$$

In both cases, you can then simplify the distributed expression to arrive at your answer. The example below, in which 5 is the outside multiplier, demonstrates that this is true. The expression on the right, which is simplified using the distributive property, is shown to be equal to 15, which is the resulting value on the left as well.

$$\begin{aligned} 5(6-3) &= 5(6)-5(3) \\ 5(3) &= 30-15 \\ 15 &= 15 \end{aligned}$$

Example	
Problem	**Rewrite the expression 20(9 – 2) using the distributive property of multiplication over subtraction. Then simplify.**
$\mathbf{20\,(9 - 2) = 20(9) - 20(2)}$	In the original expression, the 9 and the 2 are grouped in parentheses. Using arrows, you can see how the 20 is distributed to each number so that the 9 and 2 are both multiplied by 20 individually.
180 – 40 = 140	Here, the resulting product of 40 is subtracted from the product of 180, resulting in an answer of 140.
Answer	20(9 – 2) = 20(9) – 20(2) = 140

Self Check B

Rewrite the expression 10(15 – 6) using the distributive property of subtraction.

Summary

The distributive properties of addition and subtraction can be used to rewrite expressions for a variety of purposes. When you are multiplying a number by a sum, you can add and then multiply. You can also multiply each addend first and then add the products. This can be done with subtraction as well, multiplying each number in the difference before subtracting. In each case, you are distributing the outside multiplier to each number in the parentheses, so that multiplication occurs with each number before addition or subtraction occurs. The distributive property will be useful in future math courses, so understanding it now will help you build a solid math foundation.

1.8.2 Self Check Solutions

Self Check A

Rewrite the expression 30(2 + 4) using the distributive property of addition.

30(2) + 30(4)

The number 30 is distributed to both the 2 and the 4, so that both 2 and 4 are multiplied by 30.

Self Check B

Rewrite the expression 10(15 – 6) using the distributive property of subtraction.

10(15) – 10(6)

The 10 is correctly distributed so that it is used to multiply the 15 and the 6 separately.