Unit 1: Whole Numbers

1.1.1 Place Value and Names for Whole Numbers

Learning Objective(s)
1. Find the place value of a digit in a whole number.
2. Write a whole number in words and in standard form.
3. Write a whole number in expanded form.

Introduction

Mathematics involves solving problems that involve numbers. We will work with **whole numbers**, which are any of the numbers 0, 1, 2, 3, and so on. We first need to have a thorough understanding of the number system we use. Suppose the scientists preparing a lunar command module know it has to travel 382,564 kilometers to get to the moon. How well would they do if they didn't understand this number? Would it make more of a difference if the 8 was off by 1 or if the 4 was off by 1?

In this section, you will take a look at digits and place value. You will also learn how to write whole numbers in words, standard form, and expanded form based on the place values of their digits.

The Number System

Objective 1

A **digit** is one of the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, or 9. All numbers are made up of one or more digits. Numbers such as 2 have one digit, whereas numbers such as 89 have two digits. To understand what a number really means, you need to understand what the digits represent in a given number.

The position of each digit in a number tells its value, or **place value**. We can use a **place-value chart** like the one below to easily see the place value for each digit. The place values for the digits in 1,456 are shown in this chart.

Place-Value Chart														
Trillions			Billions			Millions			Thousands			Ones		
											1	4	5	6
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

In the number 1,456, the digit 1 is in the thousands place. The digit 4 is in the hundreds place. The digit 5 is in the tens place, and the digit 6 is in the ones place.

As you see above, you can tell a digit's value by looking at its position. Look at the number of digits to the right of the digit, or write your number into a place-value chart, with the last digit in the ones column. Both these methods are shown in the example below.

Example

Problem **The development of a city over the past twenty years cost $962,234,532,274,312. What is the value of the digit 6 in this number?**

Place-Value Chart														
Trillions			Billions			Millions			Thousands			Ones		
9	**6**	2	2	3	4	5	3	2	2	7	4	3	1	2
Hundreds	**Tens**	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Write the number in the place-value chart. Read the value of the 6 from the chart.

$962,234,532,274,312
60,000,000,000,000

Answer The value of the digit 6 is 60 trillion.

Self Check A
In a far away galaxy, there are 2,968,351,472 stars. What does the digit 3 represent in this problem?

Periods and Standard Form

Objective **2**

The **standard form** of a number refers to a type of notation in which digits are separated into groups of three by commas. These groups of three digits are known as **periods**. For example, 893,450,243 has three periods with three digits in each period, as shown below.

Place-Value Chart														
Trillions			Billions			Millions			Thousands			Ones		
						8	9	3	4	5	0	2	4	3
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Let's examine the number of digits and periods in a greater number. The number of body cells in an average adult human is about one hundred trillion. This number is written as 100,000,000,000,000. Notice that there are 15 digits and 5 periods. Here is how the number would look in a place-value chart.

Place-Value Chart														
Trillions			Billions			Millions			Thousands			Ones		
1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

You are now familiar with the place values of greater numbers, so let's examine a problem that involves converting from standard form to a word name.

Converting Standard Form to Word Names

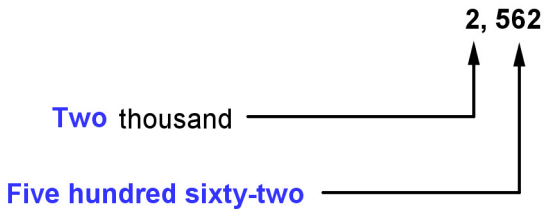
We often use word names to write numbers. A word name for 42 is "forty-two." The total number of weeks in a year, 52, is written as "fifty-two."

For whole numbers with three digits, use the word "hundred" to describe how many hundreds there are in the number. For example, for the number of days in a normal year, 365, the digit 3 is in the hundreds place. The word name for the number is "three hundred sixty-five."

For whole numbers with four digits, begin the name with the number of thousands, followed by the period name, as in the example below.

Example

Problem **A man owes $2,562 on a car. Write the word name for this.**

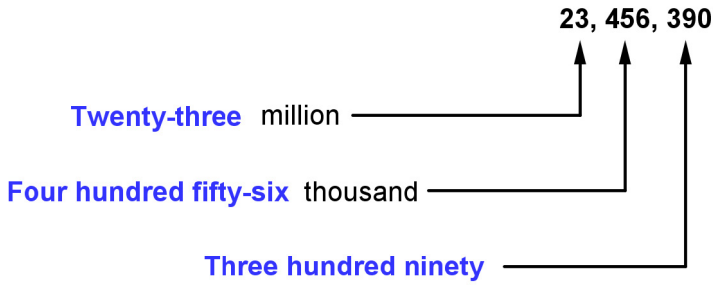


Answer The word name is two thousand, five hundred sixty-two.

For word names of greater numbers, begin at the left with the greatest period. For each period, write the one- to three-digit number in the period, and then the period name. See the example below.

Example

Problem — **The construction of a new athletic center cost $23,456,390. Write the word name for this number.**



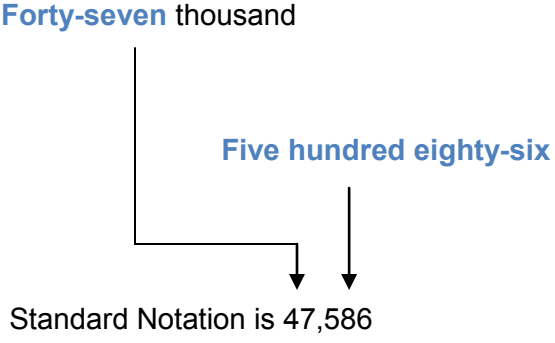
Answer The word name is twenty-three million, four hundred fifty-six thousand, three hundred ninety.

Converting Word Names to Standard Form

When converting word names to standard form, the word “thousand” tells you which period the digits are in. See the example below.

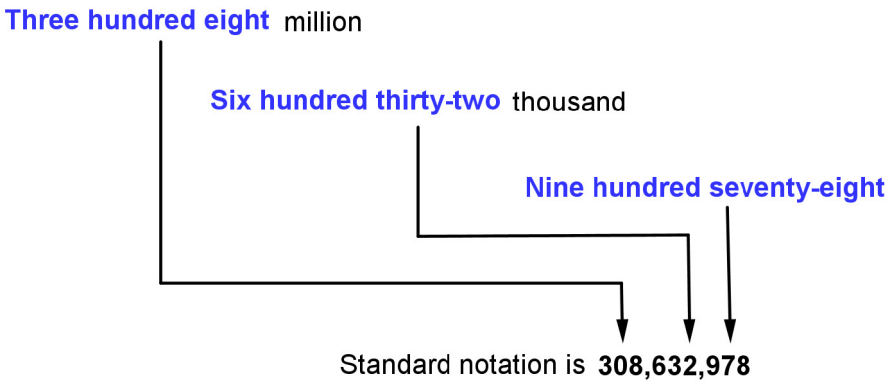
Example

Problem — **Forty-seven thousand, five hundred eighty-six blueberries are produced on a farm over the course of three years. Write this number in standard form.**



Answer The number in standard form is 47,586.

Below is an example with a number containing more digits. The words “million” and “thousand” tell you which periods the digits are in. The periods are separated by commas.

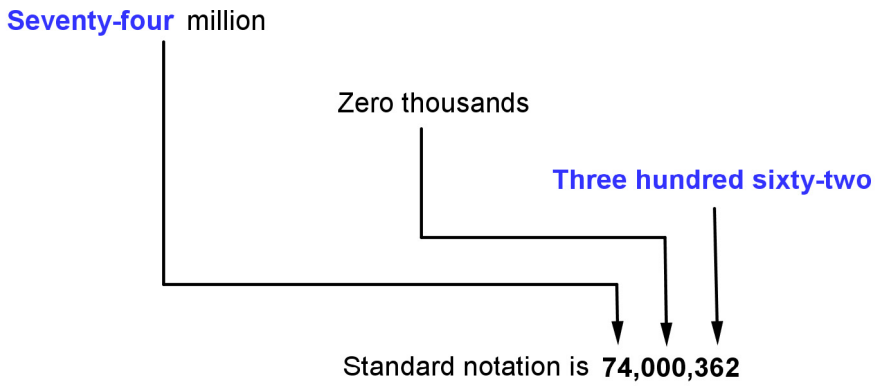
Example	
Problem	**There are three hundred eight million, six hundred thirty-two thousand, nine hundred seventy-eight bacteria in a sample of soil. Write this number in standard form.**
	
Answer	The number in standard form is 308,632,978.

Some numbers in word form may not mention a specific period. For example, three million, one hundred twelve written in standard form is 3,000,112. Because the thousands period is not mentioned, you would write three zeros in the thousands period. You can use a place-value chart to make it easier to see the values of the digits. See the example below.

Example	
Problem	**A company had a new office building constructed. The final cost was seventy-four million, three hundred sixty-two dollars. Write this number in standard form.**

Place-Value Chart															
Trillions			Billions			Millions			Thousands			Ones		
							7	4	0	0	0	3	6	2
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

Placing this number in a place- value chart shows that the thousands period is zero.



Remember to separate each period with a comma.

Answer The number written in standard form is $74,000,362.

Writing Numbers in Expanded Form

Objective 3

Sometimes it is useful to write numbers in **expanded form**. In expanded form, the number is written as a sum of the value of each digit.

Example

Problem **During the week, Mike drives a total of 264 miles. Write 264 in expanded form.**

First, identify the value of each digit.

In numerical form:

2**6**4 **200**

2**6**4 **60**

2**6**4 **4**

In word form:

264	**2 hundreds**
2**6**4	**6 tens**
26**4**	**4 ones**

Then, write the numbers as a sum.

Answer 264 written in expanded form is
200 + 60 + 4, or
2 hundreds + 6 tens + 4 ones, or
(2 • 100) + (6 • 10) + (4 • 1)

You can also use a place-value chart to help write a number in expanded form. Suppose the number of cars and pick-up trucks in the U.S. at this very moment is 251,834,697. Place this number in a place-value chart.

Place-Value Chart														
Trillions			Billions			Millions			Thousands			Ones		
						2	5	1	8	3	4	6	9	7
Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones	Hundreds	Tens	Ones

2 hundred millions	200,000,000
+ 5 ten millions	+50,000,000
+ 1 million	+1,000,000
+ 8 hundred thousands	+800,000
+ 3 ten thousands	+30,000
+ 4 thousands	+4,000
+ 6 hundreds	+600
+ 9 tens	+90
+ 7 ones	+7

Summary

Whole numbers that are greater than 9 consist of multiple digits. Each digit in a given number has a place value. To better understand place value, numbers can be put in a place-value chart so that the value of each digit can be identified. Numbers with more than three digits can be separated into groups of three digits, known as periods. Any whole number can be expressed in standard form, expanded form, or as a word name.

1.1.1 Self Check Solutions

Self Check A
three hundred thousands
The digit 3 is in the hundred thousands place.

1.1.2 Rounding Whole Numbers

Learning Objective(s)
1 Learn the rules for rounding.
2 Round whole numbers to specific place values, including tens, hundreds, and thousands.

Introduction

In some situations, you don't need an exact answer. In these cases, **rounding** the number to a specific **place value** is possible. For example, if you travelled 973 miles, you might want to round the distance to 1,000 miles, which is easier to think about. Rounding also comes in handy to see if a calculation is reasonable.

Rounding Whole Numbers

Objective 1

Objective 2

These are the rules for rounding **whole numbers**:

First, identify the digit with the place value to which you are rounding. You might circle or highlight the digit so you can focus on it better.

Then, determine the possible numbers that you would obtain by rounding. These possible numbers are close to the number that you're rounding to, but have zeros in the digits to the right.

If you are rounding 186 to the nearest ten, then180 and 190 are the two possible numbers to round to, as 186 is between 180 and 190. But how do you know whether to round to180 or 190?

Usually, round a number to the number that is closest to the original number.

When a number is halfway between the two possible numbers, round up to the greater number.

Since 186 is between 180 and 190, and 186 is closer to 190, you round up to 190.

You can use a number line to help you round numbers.

Example

Problem **A camera is dropped out of a boat, and sinks to the bottom of a pond that is 37 feet deep. Round 37 to the nearest ten.**

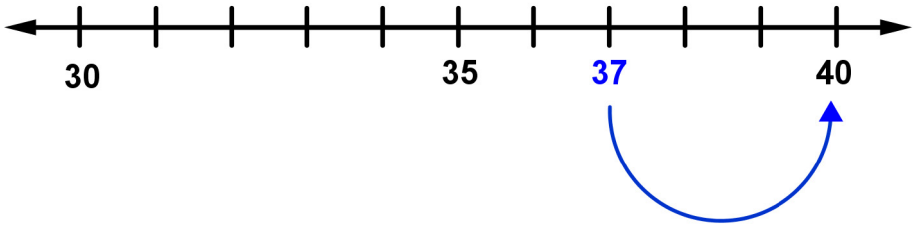
37

The digit you're rounding to is the tens digit, 3.

30 37 40

37 is between 30 and 40.

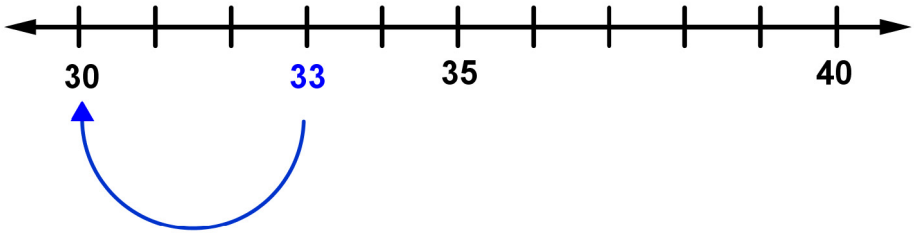
37 is only 3 away from 40, but it's 7 away from 30. So, 37 is closer to 40.



Answer To the nearest ten, 37 rounds to 40.

Example

Problem **Round 33 to the nearest ten.**



33 → 30, because 33 is closer to 30.

Answer To the nearest ten, 33 rounds to 30.

You can determine where to round without using a number line by looking at the digit to the right of the one you're rounding to. If that digit is less than 5, round down. If it's 5 or greater, round up. In the example above, you can see without a number line that 33 is rounded to 30 because the ones digit, 3, is less than 5.

Example	
Problem	***Round 77 to the nearest ten.***
77 →	80, because the ones digit, 7, is 5 or greater.
Answer	77 rounded to the nearest ten is 80.

Example	
Problem	**There are 576 jellybeans in a jar. Round this number to the nearest ten.**
576 →	580, because the ones digit, 6, is 5 or greater.
Answer	576 rounded to the nearest ten is 580.

In the previous examples, you rounded to the tens place. The rounded numbers had a 0 in the ones place. If you round to the nearest hundred, the rounded number will have zeros in the tens and ones places. The rounded number will resemble 100, 500, or 1200.

Example	
Problem	**A runner ran 1,539 meters, but describes the distance he ran with a rounded number. Round 1,539 to the nearest hundred.**
1,539 →	1,500, because the next digit is less than 5.
Answer	1,539 rounded to the nearest hundred is 1,500.

If you round to the nearest thousand, the rounded number will have zeros in the hundreds, tens, and ones places. The rounded number will resemble 1,000, 2,000, or 14,000.

Example	
Problem	**A plane's altitude increased by 2,721 feet. Round this number to the nearest thousand.**
2,721 →	3,000, because the next digit, 7, is 5 or greater.
Answer	2,721 rounded to the nearest thousand is 3,000.

Now that you know how to round to the nearest ten, hundred, and thousand, try rounding to the nearest ten thousand.

Example	
Problem	**Round 326,749 to the nearest ten thousand.**
326,749 →	330,000, because the next digit, 6, is 5 or greater.
Answer	326,749 rounded to the nearest ten thousand is 330,000.

Self Check A
A record number of 23,386 people voted in a city election. Round this number to the nearest hundred.

Summary

In situations when you don't need an exact answer, you can round numbers. When you round numbers, you are always rounding to a particular place value, such as the nearest thousand or the nearest ten. Whether you round up or round down usually depends on which number is closest to your original number. When a number is halfway between the two possible numbers, round up to the larger number.

1.1.2 Self Check Solutions

Self Check A
A record number of 23,386 people voted in a city election. Round this number to the nearest hundred.

23,400
The two possible numbers are 23,300 and 23,400, and 23,386 is closer to 23,400. The tens digit, 8, is 5 or greater so you should round up.

1.1.3 Comparing Whole Numbers

Learning Objective(s)
1 Use > or < to compare whole numbers.

Introduction

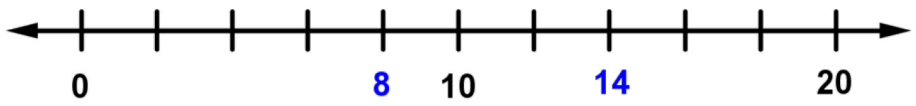
There will be times when it's helpful to compare two numbers and determine which number is greater, and which one is less. This is a useful way to compare quantities such as travel time, income, or expenses. The symbols < and > are used to indicate which number is greater, and which is less than the other.

Comparing Whole Numbers

When comparing the values of two numbers, you can use a number line to determine which number is greater. The number on the right is always greater than the number on the left. In the example below, you can see that 14 is greater than 8 because 14 is to the right of 8 on the number line.

Example

Problem **Which number is greater, 8 or 14?**



14 is to the right of 8, so 14 is greater than 8.

Answer 14 is greater than 8.

In the example below, you can determine which number is greater by comparing the digits in the ones place value.

Example	
Problem	**Which number is greater, 33 or 38?**

In both **3**3 and **3**8, the digit in the tens place is **3.**

Because they have the same number in the tens place, you can determine which one is greater by comparing the digits in the ones place.

In the number 3**8**, the digit in the ones place is **8**.

In the number 3**3**, the digit in the ones place is **3**.

Because 8 is greater than 3, 38 is greater than 33.

Answer 38 is greater than 33. This answer was obtained from comparing their digits in the ones place value, which are 8 and 3, respectively.

Self Check A
Which number is greater, 17 or 11?

If one number is significantly greater than another number, it may be difficult to compare the numbers effectively with a number line. In general, **whole numbers** with more digits are greater than whole numbers with fewer digits. For example, 542 is greater than 84 because 542 has the digit 5 in the hundreds place. There are no hundreds in 84.

Self Check B
Which number is greater, 71 or 710?

Inequalities

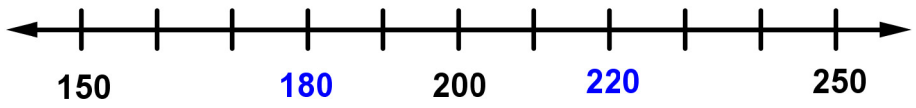
Objective 1

An **inequality** is a mathematical sentence that compares two numbers that aren't equal. Instead of an equal sign (=), inequalities use greater than (>) or less than (<) symbols. The important thing to remember about these symbols is that the small end points towards the lesser number, and the larger (open) end is always on the side of the greater number.

There are other ways to remember this. For example, the wider part of the symbol represents the jaws of an alligator, which "gobbles up" the greater number. So "35 is greater than 28" can be written as 35 > 28, and "52 is less than 109" can be written as 52 < 109.

Example

Problem — **Replace ? with < or > to make a true sentence: 180 ? 220.**



180 is to the left of 220, so 180 < 220. The symbol points at 180, which is the lesser number.

Answer 180 < 220

Self Check C

Which expression correctly compares the numbers 85 and 19?

A) 85 < 19 B) 19 = 85 C) 85 > 19 D) 19 > 85

Many times an answer needs to be a range of values rather than just a single value. For example, you want to make more than $22 an hour. This can be expressed as all numbers greater than 22. See the example below.

Example

Problem — **? > 22. What whole number(s) will make this statement true?**

The symbol points at 22, so the numbers you want to put in the brackets are *greater* than 22. There are many numbers that work.

Answer 23, 24, 25, 26, and any additional whole numbers that are greater than 26 make this statement true.

Self Check D

A farmer has produced 230 pumpkins for the autumn harvest. Last year, he produced 198. Write an expression that compares these two numbers.

Summary

To compare two values that are not the same, you can write an inequality. You can use a number line or place value to determine which number is greater than another number. Inequalities can be expressed using greater than (>) or less than (<) symbols.

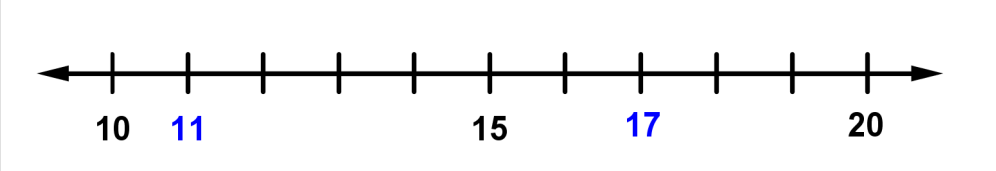
1.1.3 Self Check Solutions

Self Check A

Which number is greater, 17 or 11?

17

The number 17 is 6 units to the right of 11 on the number line.



Number line for Self Check A.

Self Check B

Which number is greater, 71 or 710?

710

The number 710 has 7 hundreds, but 71 has no hundreds.

Self Check C

Which expression correctly compares the numbers 85 and 19?

85 > 19

The open part of the symbol faces the larger number, 85, and the symbol points at the smaller number, 19.

Self Check D

A farmer has produced 230 pumpkins for the autumn harvest. Last year, he produced 198. Write an expression that compares these two numbers.

230 > 198

230 is greater than 198, and this is reflected in the symbol because the open part of the symbol faces 230. The expression 198 < 230 would also be correct.